



**Coimisiún na Scrúduithe Stáit**  
**State Examinations Commission**

**Junior Certificate 2013**

**Marking Scheme**

**Mathematics**  
**(Project Maths – Phase 3)**

**Higher Level**

## **Note to teachers and students on the use of published marking schemes**

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

## **Future Marking Schemes**

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Junior Certificate Examination, 2013

Mathematics  
(Project Maths – Phase 3)

Paper 1

Higher Level

**Model Solutions – Paper 1**

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

**Question 1****(15 marks)**

- (a) (i) The columns in the table below represent the following sets of numbers:  
Natural numbers ( $\mathbb{N}$ ), Integers ( $\mathbb{Z}$ ), Rational numbers ( $\mathbb{Q}$ ),  
Irrational numbers ( $\mathbb{R} \setminus \mathbb{Q}$ ) and Real numbers ( $\mathbb{R}$ ).

Complete the table by writing either 'Yes' or 'No' into each box indicating whether each of the numbers  $\sqrt{5}$ , 8, -4,  $3\frac{1}{2}$ ,  $\frac{3\pi}{4}$  is or is not an element of each.

(One box has already been filled in. The 'Yes' indicates that the number 8 is an element of the set of Real numbers,  $\mathbb{R}$ ).

Number/Set	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$(\mathbb{R} \setminus \mathbb{Q})$	$\mathbb{R}$
$\sqrt{5}$	No	No	No	Yes	Yes
8	Yes	Yes	Yes	No	Yes
-4	No	Yes	Yes	No	Yes
$3\frac{1}{2}$	No	No	Yes	No	Yes
$\frac{3\pi}{4}$	No	No	No	Yes	Yes

- (ii) In the case of  $\sqrt{5}$  explain your choice in relation to the set of Irrational numbers ( $\mathbb{R} \setminus \mathbb{Q}$ ) (i.e. give a reason for writing either 'Yes' or 'No').

$\sqrt{5}$  cannot be written as a fraction

- (b) Use the properties of surds to show that  $\sqrt{98} - \sqrt{18} + \sqrt{2}$  simplifies to  $5\sqrt{2}$ .

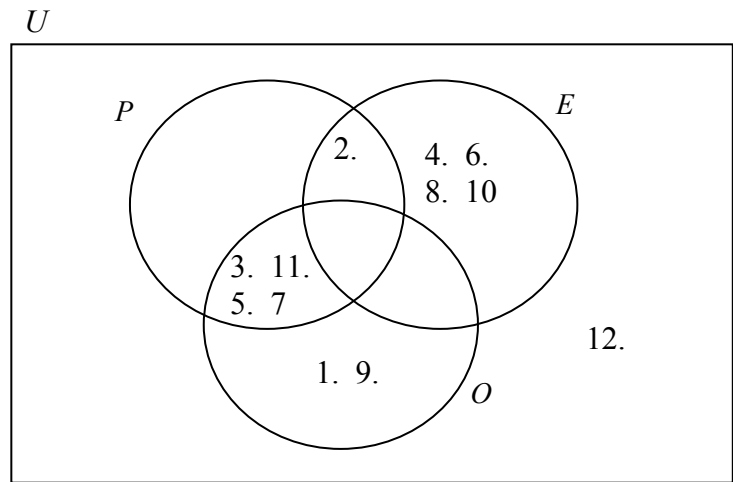
$$7\sqrt{2} - 3\sqrt{2} + \sqrt{2} = 5\sqrt{2}$$

**Question 2**

**(25 marks)**

$U = \{1, 2, 3, \dots, 12\}$ .  $P$  is the set of prime numbers less than 12.  $E$  is the set of even numbers less than 12.  $O$  is the set of odd numbers less than 12.

(a) Represent these sets on the Venn diagram.



(b) Name any set on this diagram (after part (a) has been completed) that is a null set.

$P \setminus (E \cup O)$ ,  $P \cap E \cap O$ ,  $E \cap O$ ,  $(E \cap O) \setminus P$

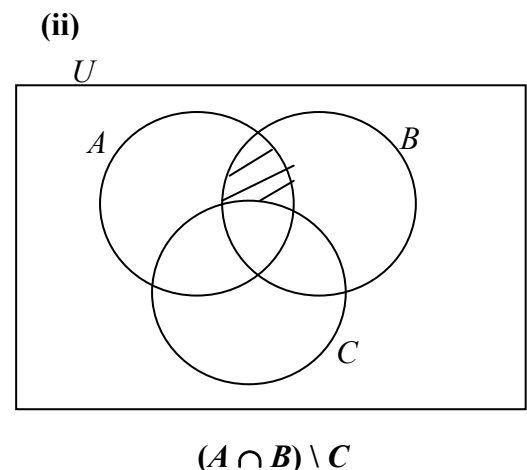
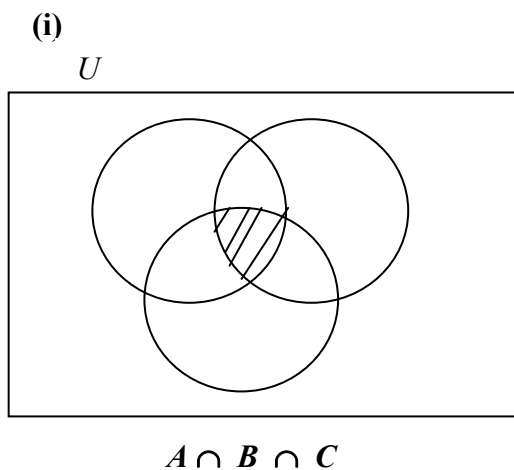
(c) If a number is drawn at random from set  $P$ , what is the probability that it is even?

$\frac{1}{5}$

**Question 3**

**(40 marks)**

(a) For diagrams (i) and (ii) below, shade in the named region.



- (b) The box on the right contains six statements,  
(note:  $P'$ , is the complement of a set  $P$ ).

A number of the statements are incorrect.

Write down one incorrect statement.

(iv)  $A \setminus B = B \setminus A$  or (iii)  $(A \setminus B) \setminus C = A \setminus (B \setminus C)$

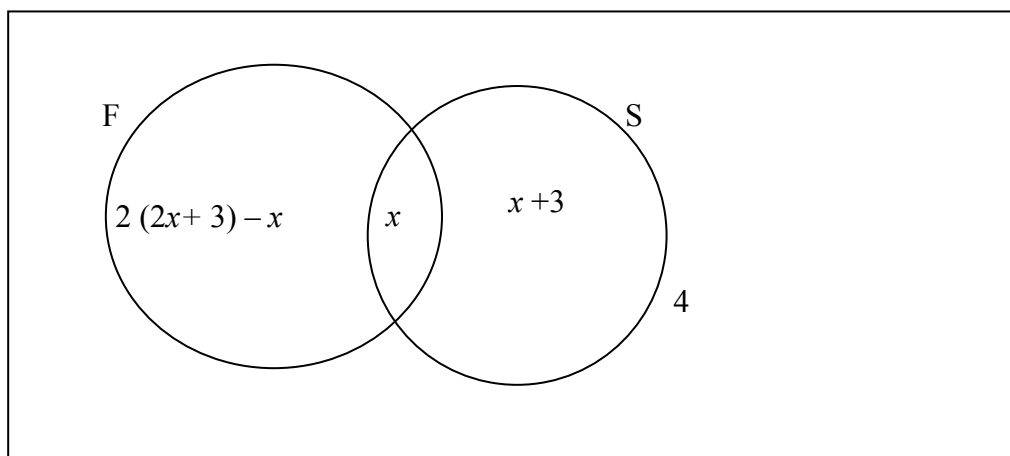
Statements	
(i)	$A \cup B = B \cup A$
(ii)	$(A \cup B) \cup C = A \cup (B \cup C)$
(iii)	$(A \setminus B) \setminus C = A \setminus (B \setminus C)$
(iv)	$(A \cap B)' = U \setminus (A \cap B)$
(v)	$A \setminus B = B \setminus A$
(vi)	$B \setminus (A \cup C) = (B \cup C) \setminus A \setminus C$

Draw a diagram or give an example to explain your choice.

Diagram or explanation

- (c) A group of 38 students were asked if they had ever been to France or Spain.  
The number who had been to Spain only was 3 more than the number who had been to both countries.  
Twice as many had been to France as Spain.  
4 students had not been to either country.

Find how many had been to both countries.



$$2(2x+3)-x + x + x+3 = 34 \Rightarrow x = 5$$



**Question 4****(35 marks)**

The minimum wage per hour for different categories of workers is shown in the table.

By law the Under 18 minimum wage is set at 70% of the minimum wage for an experienced adult worker.

- (a) Verify that this is true for the rates shown in the table on the right.

Category	Min. Wage per hour
Experienced adult worker	€8.65
Aged under 18	€6.06
Over 18 in first year from date of first employment	€6.92
Over 18 in second year of first employment	€7.79
Source: <a href="http://www.citizensinformation.ie">www.citizensinformation.ie</a>	

$8.65 \times 0.7 = 6.055$  which is 6.06 correct to two dps.  
Or any other check.

- (b) The government has decided that it is going to reduce all minimum wage rates by 6%. Calculate the new minimum wage for an experienced adult worker, correct to two decimal places, after this reduction.

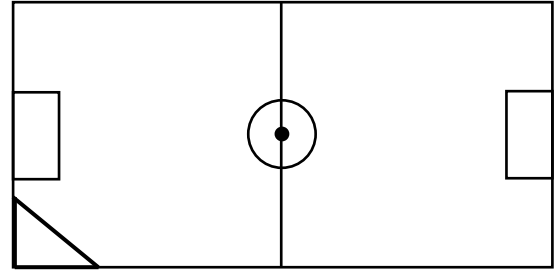
$$8.65 \times 0.94 = €8.13$$

- (c) John is an experienced adult worker. After the reduction he says “If the minimum wage were to be increased by 6% then I would be back earning €8.65 per hour.” Is John’s statement correct? Explain your answer.

No.  
 $8.13 \times 1.06 = €8.62$   
This is not as high as the original starting point.

**Question 5****(10 marks)**

Two members of a soccer club want to find out if their football pitch has been lined out properly.



- (a) They have a 10 metre tape measure, a calculator, pen and paper.  
By using only these, explain how they could test if the angle at each corner is a right angle.

Start at the corner flag. Use the tape measure to measure a certain distance out along the side-line. e.g. 5 m.  
Then measure a certain distance out along the goal-line. e.g. 4 m.  
Then measure the distance between these two end points  
Using Pythagoras Theorem see if the calculated distance is the same as the measured distance.

- (b) By using only a trundle wheel, calculator, pen and paper, explain how the two members could test if the 'centre-circle' on the pitch is really a circle. (You may assume that the centre spot on the pitch is the centre of the circle).

Use the trundle wheel to measure the radius. i.e distance from the centre spot to anywhere on the circumference.  
Use Circumference =  $2\pi r$  to calculate the circumference.  
Then use the trundle wheel to measure the circumference on the circle and see if the two match.

**Question 6****(15 marks)**

Car A and Car B set off from a starting point  $S$  at the same time. They travel the same route to destination  $D$ , which is 70 km away. Car A travels at an average speed of 50 km/h and car B travels at an average speed of 45 km/h.

How far will car B have travelled by the time car A arrives at destination  $D$ ?



**Car A:** (Time to reach D)  $T = D/S = 70/50 = 1.4$  h

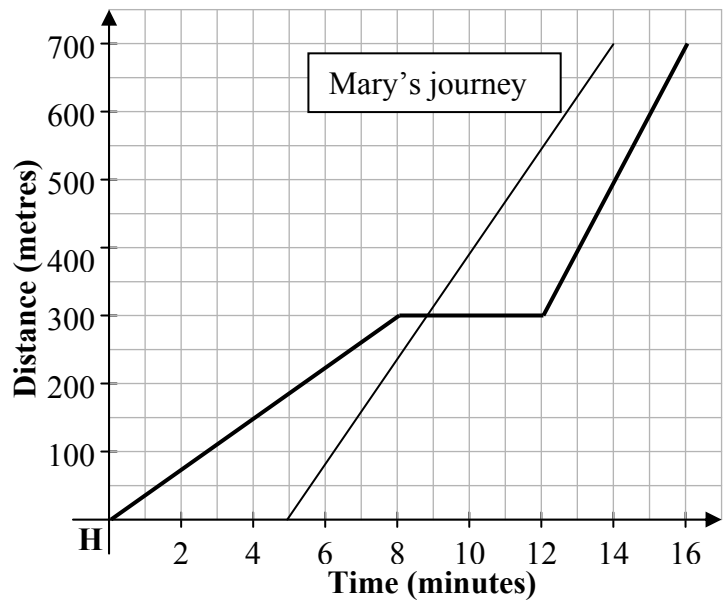
**Car B:** Distance travelled  $45 \times 1.4 = 63$  km

**Question 7**

**(20 marks)**

Angela leaves home (H) at 5 pm to go to football practice, which is 700 m away. The graph shows her journey, on foot, to football practice.

- (a) One of the stories below matches Angela's journey. Place a tick in the box beside the correct matching story. (Note: Only **one** story matches Angela's journey).



Story	Tick one story (✓)
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at a slower pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.12 for four minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for five minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at a faster pace and arrives at practice at 5.16.	✓
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at the same pace and arrives at practice at 5.16.	

- (b) Mary also lives 700 m from football practice, but cycles to practice. She leaves home five minutes after Angela. She cycles at a constant pace and arrives at practice two minutes before Angela. Represent Mary's journey on the graph above.

**Question 8**

**(30 marks)**

(a) Express in its simplest form:  $\frac{5-x}{5} + \frac{x-4}{4}$ .

$$\frac{4(5-x) + 5(x-4)}{20} = \frac{x}{20}$$

(b) Solve for  $x$ :  $3x^2 + 11x = 4$ .

$$\begin{aligned} 3x^2 + 11x - 4 &= 0 \\ (3x-1)(x+4) &= 0 \\ x = \frac{1}{3} \quad x &= -4 \end{aligned}$$

$$\begin{aligned} 3x^2 + 11x - 4 &= 0 \\ x &= \frac{-11 \pm \sqrt{11^2 - 4(3)(-4)}}{2(3)} \\ x &= \frac{-11 \pm 13}{6} \\ x = \frac{1}{3} \quad x &= -4 \end{aligned}$$

(c) Divide  $2x^3 + x^2 - 13x + 6$  by  $x + 3$ .

Method A

$$\begin{array}{r} 2x^2 - 5x + 2 \\ x+3 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 + 6x^2} \phantom{+ 6} \\ -5x^2 - 13x \phantom{+ 6} \\ \underline{-5x^2 - 15x} \phantom{+ 6} \\ 2x + 6 \\ \underline{2x + 6} \\ 0 \end{array}$$

Method B

	$ax^2$	$bx$	$c$
$x$	$ax^3$	$bx^2$	$cx$
$+3$	$3ax^2$	$3bx$	$3c$

$$\begin{aligned} ax^3 = 2x^3 &\Rightarrow a = 2 \\ \text{-----} \\ x^2(3a+b) = -5 &\Rightarrow 3a+b = -5 \\ \Rightarrow 6+b = -5 &\Rightarrow b = -11 \\ \text{-----} \\ 3c = 6 &\Rightarrow c = 2 \end{aligned}$$

- (d) A company employs two drivers, John and David. Each has use of a company car and small van. The company buys €30 worth of Toll Tags for each driver. Each time that a vehicle goes through the M50 Toll, a charge will be deducted from the Toll Tags.

John goes through the M50 toll five times in his car and four times in his small van. He then has €7.90 remaining on his Toll Tags. David goes through the M50 Toll twice in his car and six times in his small van. He then has €8.40 left on his Toll Tags.

Calculate how much it costs for a car and for a small van to go through the M50 Toll.

$$\begin{aligned}5x + 4y &= 30 - 7 \cdot 90 = 22 \cdot 10 \\2x + 6y &= 30 - 8 \cdot 0 = 21 \cdot 60 \\ \Rightarrow x &= €2 \cdot 10 \\ y &= €2 \cdot 90\end{aligned}$$

### Question 9

(15 marks)

The 'Multiplier' is a variable used by economists to measure the affect of an increase in spending in an economy.

One version of the Multiplier is  $M = \frac{1}{S+P}$  where  $M$  is the Multiplier,  $S$  relates to savings and  $P$  relates to imports.

- (a) Calculate the value of the  $M$ , the Multiplier, if  $S = 0.2$  and  $P = 0.1$ .

$$\left( \frac{1}{0.2 + 0.1} \right) = 3\frac{1}{3} \text{ or } \frac{10}{3} \text{ or } 3.3$$

- (b) Explain the effect on the size of  $M$  if the value of  $P$  increases.

Denominator increases so value of fraction **decreases**.

- (c) Sometimes the above formula is used to calculate  $P$ . Rearrange the formula to make  $P$  its subject.

$$M = \frac{1}{S+P}$$
$$MS + MP = 1$$
$$MP = 1 - MS$$
$$P = \frac{1-MS}{M} \text{ or } P = \frac{1}{M} - S$$

**Question 10**

**(20 marks)**

- (a) If  $n = 7$  find the value of  $2n$  and also the value of  $2n + 1$ .

$$2(7) = 14$$
$$2(7) + 1 = 15$$

- (b) (i)  $x$  represents an even number. Explain why  $x + 2$  is the next even number.

2 is the lowest even no. so adding 2 on to an even no. will give the next even no.

- (ii) If one third of the smaller even number is subtracted from half of the larger even number the result is 8. Find the value of  $x$ .

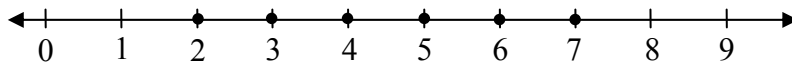
$$\frac{x+2}{2} - \frac{x}{3} = 8$$
$$x = 42$$

**Question 11****(20 marks)**

(a) Solve the following inequality and show the solution on the number line.

$$-2 \leq \frac{1}{2}x - 3 < 1, x \in \mathbb{N}.$$

$$2 \leq x < 8$$



(b) Josephine hopes to go to college. She has saved €3000. She will attend college for 32 weeks in her first year. She plans to have at least €800 left at the end of the year.

(i) If she spends € $x$  each week, write an inequality to represent her spending during the year.

$$32x \leq 3000 - 800 \text{ or similar}$$

(ii) Hence, or otherwise, find the maximum amount Josephine can spend each week.

$$32x \leq 2200$$

$$x \leq \text{€}68.75$$

**Question 12**

**(10 marks)**

Irish Sport Promotions has designed a company logo. The actual size of the logo is shown here.

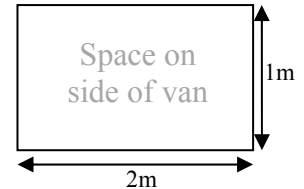
- (a) Write the dimensions of the logo, to the nearest mm, below.



**Width**   27   mm    **Height**   15   mm

- (b) The company wants to enlarge this logo in order to put it on the side of its vans.

The space available for the logo on the side of each van is:  
width 2 m and height 1 m.



If the company wants to keep the same width to height ratio as in the original logo, calculate the dimensions of the largest logo that will fit onto the side of the van.

Make height of logo = 1000 mm

$$\frac{15}{27} = \frac{1000}{x}$$

$x = 1800$  mm (= width) (or 1.8 m) (or 180 cm)

OR

Make height of logo = 1 m

$$\frac{15}{27} = \frac{1}{x}$$

$x = 1.8$  m

Scale Factor =  $\frac{1000}{15} = 66.6$   
 $27 \times 66.66 = 1799.82$  mm

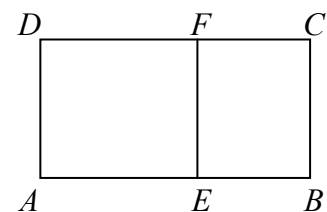
**Question 13**

**(10 marks)**

$ABCD$  is a rectangle.  $AEFD$  is a square.  $|AD| = 1$  cm.  $|DC| = x$  cm.

- (i) Write  $|EB|$  in terms of  $x$ .

$$x - 1$$



- (ii) If  $\frac{|AB|}{|AD|} = \frac{|EF|}{|EB|}$ , find the distance  $x$ . Give your answer correct to two decimal places.

$$\frac{x}{1} = \frac{1}{x-1}$$

$$x(x-1) = 1 \Rightarrow x^2 - x - 1 = 0$$

$$x = \frac{1 \pm \sqrt{5}}{2};$$

$x = 1.618... = 1.62$  cm (discard neg. value)



**Question 14**

**(15 marks)**

Investigate whether the pattern in the table below is linear, quadratic or exponential.  
 Explain your conclusion.

Term 1	Term 2	Term 3	Term 4	Term 5
$2a - b + 2c$	$8a - 2b + 2c$	$18a - 3b + 2c$	$32a - 4b + 2c$	$50a - 5b + 2c$

$2a - b + 2c$	}			
$8a - 2b + 2c$	}	Diff = $6a - b$		
$18a - 3b + 2c$	}	Diff = $10a - b$	}	Diff = $4a$
$32a - 4b + 2c$	}	Diff = $14a - b$		
$50a - 5b + 2c$	}	Diff = $18a - b$		
$2^{\text{nd}}$ difference is constant therefore the relationship is quadratic				

**Question 15**

**(20 marks)**

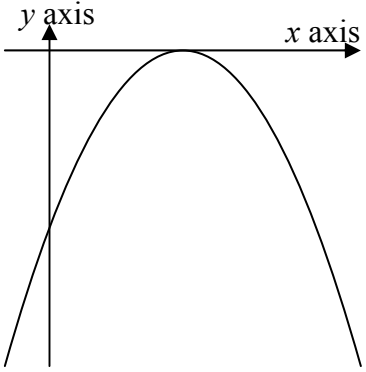
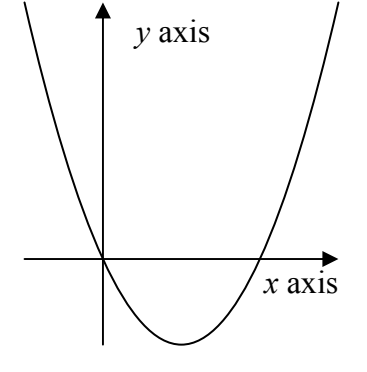
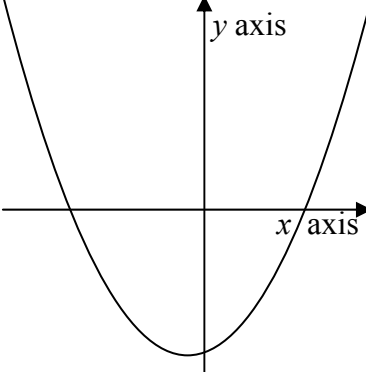
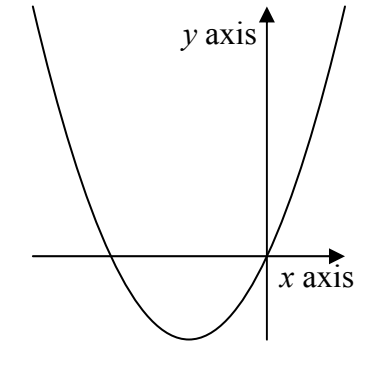
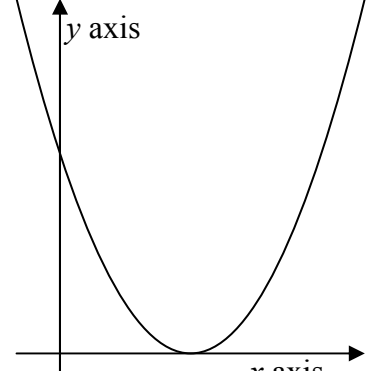
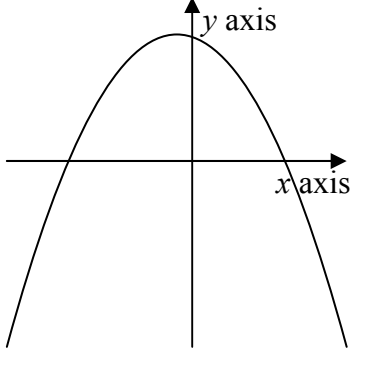
(a) Three functions:  $f(x)$ ,  $g(x)$  and  $h(x)$  are defined as follows:

$$f(x) = 2x^2 + x - 6, \quad g(x) = x^2 - 6x + 9 \quad \text{and} \quad h(x) = x^2 - 2x.$$

Solve $f(x) = 0$	Solve $g(x) = 0$	Solve $h(x) = 0$
<div style="border: 1px solid black; padding: 5px; margin: 5px auto; width: 80%;"> <math display="block">(2x - 3)(x + 2) = 0</math> <math display="block">x = \frac{3}{2}, \quad x = -2</math> </div>	<div style="border: 1px solid black; padding: 5px; margin: 5px auto; width: 80%;"> <math display="block">(x - 3)(x - 3) = 0</math> <math display="block">x = 3</math> </div>	<div style="border: 1px solid black; padding: 5px; margin: 5px auto; width: 80%;"> <math display="block">x(x - 2) = 0</math> <math display="block">x = 0, \quad x = 2</math> </div>

- (b) The table below shows the sketches of six different functions. Three of the sketches belong to the three functions from part (a).

Write  $f(x)$ ,  $g(x)$  or  $h(x)$  into the box underneath the correct sketch for each of the three functions.

<p style="text-align: center;">Diagram 1</p>  <div style="border: 1px solid black; width: 100%; height: 20px; margin-top: 10px;"></div>	<p style="text-align: center;">Diagram 2</p>  <div style="border: 1px solid black; width: 100%; height: 20px; margin-top: 10px; text-align: center;"><math>h(x)</math></div>	<p style="text-align: center;">Diagram 3</p>  <div style="border: 1px solid black; width: 100%; height: 20px; margin-top: 10px; text-align: center;"><math>f(x),</math></div>
<p style="text-align: center;">Diagram 4</p>  <div style="border: 1px solid black; width: 100%; height: 20px; margin-top: 10px;"></div>	<p style="text-align: center;">Diagram 5</p>  <div style="border: 1px solid black; width: 100%; height: 20px; margin-top: 10px; text-align: center;"><math>g(x)</math></div>	<p style="text-align: center;">Diagram 6</p>  <div style="border: 1px solid black; width: 100%; height: 20px; margin-top: 10px;"></div>

## Structure of the marking scheme Paper 1

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5	
10 mark scale	0, 10	0, 5, 10	0, 4, 8, 10	0, 4, 8, 9, 10
15 mark scale		0, 7, 15	0, 7, 12, 15	0, 5, 10, 12, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

### Marking scales – level descriptors

#### *A-scales (two categories)*

- incorrect response (no credit)
- correct response (full credit)

#### *B-scales (three categories)*

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

#### *C-scales (four categories)*

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk.

Thus, for example, *scale 10C\** indicates that 9 marks may be awarded.

## Summary of mark allocations and scales to be applied Paper 1

<b>Question 1</b> (a)(i) + (ii) 5C (b) 10B	<b>Question 2</b> (a) 5C (b) 10A (c) 10A	<b>Question 3</b> (a) 10A, 10A (b) 15C (c) 5C	<b>Question 4</b> (a) 15B (b) 10B* (c) 10C
<b>Question 5</b> (a) 5B (b) 5B	<b>Question 6</b> 15C*	<b>Question 7</b> (a) 10A (b) 10B	<b>Question 8</b> (a) 5C (b) 10C (c) 5B (d) 10D
<b>Question 9</b> (a) 5C (b) + (c) 10C	<b>Question 10</b> (a) 10B (b)(i) 5C (ii) 5C	<b>Question 11</b> (a) 5C* (b)(i) 5B (b)(ii) 10B	<b>Question 12</b> (a) + (b) 10C*
<b>Question 13</b> (i) 5A (ii) 5D*	<b>Question 14</b> 15C	<b>Question 15</b> (a) 10D (b) 10C	

## Detailed Marking Notes Paper 1

The \* for units to be applied only if answered fully correct. The \* to be applied once only per question.

The \* penalty is not applied to currency solutions.

### Question 1

(a)(i)+(ii) Scale 5C

*High Partial Credit:*

- Part (i) or (ii) fully correct

*Low Partial Credit:*

- Any one entry correct in (i)
- Some non-terminating or non-recurring decimal in (ii)

(b) Scale 10B

*Partial Credit:*

- One or two of  $7\sqrt{2}$  or  $3\sqrt{2}$  written
- Equates decimal version of both

### Question 2

(a) Scale 5C

*High Partial Credit:*

- Nine, ten or eleven elements correctly placed

*Low Partial Credit:*

- One to eight elements placed correctly

(b) Scale 10A

(c) Scale 10A

### Question 3

(a) (i) Scale 10A

(ii) Scale 10A

(b) Scale 15C

*High Partial Credit:*

- Statement correct and one side of diagram correct

*Low Partial Credit:*

- Statement correct and diagram incorrect or no diagram
- Statement incorrect but one side of diagram (or both sides) correct

(c) Scale 5C

*High Partial Credit:*

- All expressions correct [Not looking for these to be connected to the 38]

*Low Partial Credit:*

- Any one element correctly identified

### Question 4

(a) Scale 15B

*Partial Credit:*

- Any correct use of the 70% on 8.65 or 6.06

**NOTE:** 6.055 without work full marks

(b) Scale 10B\*

*Partial Credit:*

- Writes 0.94 or 94%
- Any correct use of the 6% on any wage in the table

(c) Scale 10C

*High Partial Credit:*

- Calculates €8.62 but no explanation or incorrect explanation

*Low Partial Credit:*

- Writes 1.06 or 106%
- Some correct work on the 6%

**NOTE:** Could give full answer without calculations for full marks

### Question 5

(a) Scale 5B

*Partial Credit:*

- Some correct use of the items

(b) Scale 5B

*Partial Credit:*

- Some mention of attempting to measure the circumference or measuring the radius
- Some correct use of the items

### Question 6

Scale 15C\*

*High Partial Credit:*

- Calculates the 1.4 h

*Low Partial Credit:*

- $T=D/S$  or  $D=S \times T$  written

### Question 7

(a) Scale 10A

(b) Scale 10B

*Partial Credit:*

- Any straight line, with positive slope, that goes from  $(k, 0)$  to  $(t, 700)$ , for any  $k, t$  on diagram
- Starting point marked correctly i.e.  $(5, 0)$
- End point marked correctly i.e.  $(14, 700)$

### Question 8

(a) Scale 5C

*High Partial Credit:*

- $$\frac{4(5-x) + 5(x-4)}{20}$$

*Low Partial Credit:*

- Writes any one or two of 20,  $4(5-x)$  or  $5(x-4)$

- (b) Scale 10C  
*High Partial Credit:*
- Both factors correct [Not looking for “= 0” ]
  - Full correct substitution into the  $-b$  formula

- Low Partial Credit:*
- Writes  $3x^2+11x-4 = 0$
  - Writes down the  $-b$  formula

- (c) Scale 5B  
*Partial Credit:*
- Any one term of the answer correctly identified  
 i.e.  $2x^2$  or  $+2$
  - Inserts the  $2x^3$  or the 6 into Method B

- (d) Scale 10D  
*High Partial Credit:*
- One of  $x = 2.10$  or  $y = 2.90$  calculated
- Partial Credit:*
- $5x+4y = 30-7.9$  ( $22.10$ ) and  $2x+6y = 30-8.40$  ( $21.60$ )
- Low Partial Credit:*
- Identifies the  $5x+4y$  or  $2x+6y$
  - Writes  $30-7.90$  ( $22.10$ ) or  $30-8.40$  ( $21.60$ )

### Question 9

- (a) Scale 5C  
*High Partial Credit:*
- Full correct substitution into  $\frac{1}{0.2+0.1}$
- Low Partial Credit:*
- Some correct substitution into  $\frac{1}{0.2+0.1}$

**NOTE:**  $3\frac{1}{3}$ , 3.33 or 3.3  
 without work full marks

- (b) + (c) Scale 10C  
*High Partial Credit:*
- Part (b) or (c) fully correct
- Low Partial Credit:*
- Some correct explanation of change in value in (b)  
 e.g. “denominator gets bigger”
  - One correct manipulation in (c)



### Question 10

(a) Scale 10B

*Partial Credit:*

- Calculates one of the 14 or the 15
- Some correct substitution into  $2n$  or  $2(n+1)$  or both

(b) (i) Scale 5C

*High Partial Credit:*

- One element identified correctly

*Low Partial Credit:*

- Partly correct explanation

(ii) Scale 5C

*High Partial Credit:*

- $\frac{x+2}{2} - \frac{x}{3} = 8$
- Calculates  $x = -54$  using  $\frac{x}{3} - \frac{x+2}{2} = 8$

*Low Partial Credit:*

- $\frac{x}{3} - \frac{x+2}{2} = 8$
- Writes  $\frac{x}{3}$  or  $\frac{x+2}{2}$  or both

### Question 11

(a) Scale 5C\*

*High Partial Credit:*

- $2 \leq x < 8$  [allow  $2 < x < 8$  or  $2 < x \leq 8$ ]

*Low Partial Credit:*

- Any correct manipulation

**NOTE:** The \* applied if the 8 is shaded on the numberline

(b) (i) Scale 5B

*Partial Credit:*

- $32x$  or  $3000-800$

(ii) Scale 10B

*Partial Credit:*

- Any correct manipulation of candidate's inequality from (b)(i)

### Question 12

(a) + (b) Scale 10C\*

*High Partial Credit:*

- Part (a) or (b) fully correct
- One of 27 or 15 written [tolerance of  $\pm 1$ mm]

**NOTE:** Width 15mm and height 27mm accepted in (a)

*Low Partial Credit:*

- $\frac{15}{27} = \frac{1}{x}$  or  $15:27 = 1000:x$  or  $\frac{15}{27} = \frac{1000}{x}$  in (b)
- $\frac{15}{27}$  or  $15:27$  in (b)
- Identifies the height as being 1 m (or 100 cm or 1000 mm) in (b)

**NOTE:** Correct answer without work in (b) LPC

### Question 13

(i) Scale 5A

(ii) Scale 5D\*

*High Partial Credit:*

- Full correct substitution into  $\frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

*Partial Credit:*

- $x^2 - x - 1 = 0$
- Full correct substitution into  $\frac{x}{1} = \frac{1}{x-1}$

*Low Partial Credit:*

- Some correct substitution into  $\frac{x}{1} = \frac{1}{x-1}$

**NOTE:** The “ $x-1$ ” is the candidate’s answer to (i)

### Question 14

Scale 15C

*High Partial Credit:*

- All first and second differences correct but conclusion incorrect or no conclusion

*Low Partial Credit:*

- Any one first difference correct

### Question 15

(a) Scale 10D

*High Partial Credit:*

- Any two of  $f(x) = 0$ ,  $g(x) = 0$  or  $h(x) = 0$  correctly solved

*Partial Credit:*

- Any one of  $f(x) = 0$ ,  $g(x) = 0$  or  $h(x) = 0$  correctly solved

*Low Partial Credit:*

- Writes any one of  $2x^2+x-6 = 0$ ,  $x^2-6x+9=0$  or  $x^2-2x = 0$
- Verifies one root of  $f(x)$  and/or  $h(x)$

(b) Scale 10C

*High Partial Credit:*

- Any two of  $f(x)$ ,  $g(x)$  or  $h(x)$  correctly identified

*Low Partial Credit:*

- Any one of  $f(x)$ ,  $g(x)$  or  $h(x)$  correctly identified

**NOTE:** If a function is identified (written) in two or more places, the answer is not valid





Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Junior Certificate Examination, 2013

Mathematics  
(Project Maths – Phase 3)

Paper 2

Higher Level

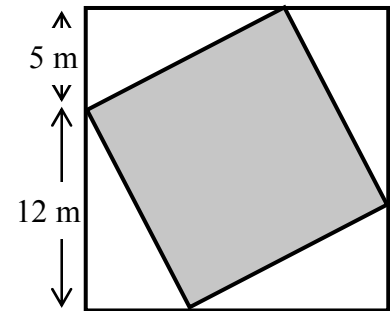
**Model Solutions – Paper 2**

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

### Question 1

20 marks

Liam's garden is in the shape of a square. It has four equal right-angled triangular lawns and a smaller square patio in the centre, as shown.



- (a) Find the length of the hypotenuse of one of the right angled triangular lawns.

$$h^2 = a^2 + b^2$$

$$\text{Length} = \sqrt{12^2 + 5^2} = 13 \text{ m}$$

- (b) Find the area of one of the triangular lawns.

$$\text{Area} = \frac{1}{2}(12)(5) = 30 \text{ m}^2$$

- (c) Find the area of the square patio in the middle.

$$\text{Area of patio} = 13^2 = 169 \text{ m}^2$$

**or**

$$\text{Area of patio} = 17^2 - (4 \times 30) = 169 \text{ m}^2$$

- (d) The patio is to be paved with rectangular flagstones of length 80cm and width 50cm. Calculate the number of flagstones Liam needs to buy to cover the patio, allowing an extra 20% for waste.

$$\begin{aligned} \text{Area of flagstone} &= 0.8 \times 0.5 = 0.4 \text{ m}^2 \\ \text{Number of flagstones} &= \frac{169}{0.4} = 422.5 \quad (\text{or } 423) \\ \text{Extra 20\%} &= 422.5 \times 0.2 = 84.5 \quad \text{or} \quad 120\% = 422.5 \times 1.2 \\ \text{Total number of flagstones} &= 507 \\ &\text{or} \\ \text{Extra 20\%} &= 423 \times 0.2 = 84.6 \quad \text{or} \quad 120\% = 423 \times 1.2 \\ \text{Total number of flagstones} &= 507.6 \\ &\text{or} \\ \text{Area to cover} &= 169 \times 1.2 = 202.8 \\ \text{Total number of flagstones} &= \frac{202.8}{0.4} = 507 \end{aligned}$$

**Question 2**

**30 marks**

The ages of the 30 people who took part in an aerobics class are as follows:

18	24	32	37	9	13	22	41	51	49
15	42	37	58	48	53	27	54	42	24
33	48	56	17	61	37	63	45	20	39

The ages of the 30 people who took part in a swimming class are as follows:

16	22	29	7	36	45	12	38	52	13
33	41	24	35	51	8	47	22	14	24
42	62	15	24	23	31	53	36	48	18

(a) Represent this data on a back-to-back stem-and-leaf diagram.

Aerobics class										Swimming class									
						9	0	7	8										
			8	7	5	3	1	2	3	4	5	6	8						
		7	4	4	2	0	2	2	2	3	4	4	4	9					
	9	7	7	7	3	2	3	1	3	5	6	6	8						
9	8	8	5	2	2	1	4	1	2	5	7	8							
		8	6	4	3	1	5	1	2	3									
					3	1	6	2											
										<b>Key: 1   5 means 15</b>									

(b) Use your diagram to identify the median in each case.

Aerobic median:  $\frac{37 + 39}{2} = 38$

Swimming median:  $\frac{29 + 31}{2} = 30$

(c) What other measure of central tendency could have been used when examining this data?

Mean or Mode

(d) Based on the data make one observation about the ages of the two groups.

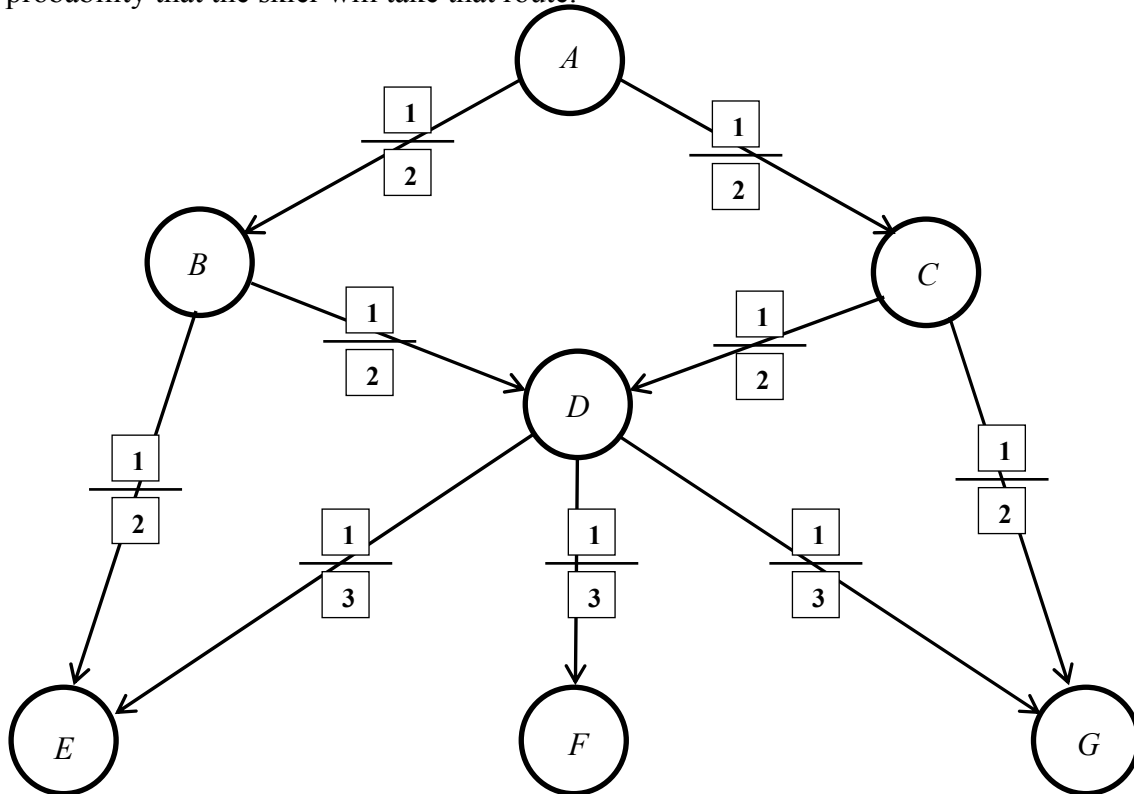
An older age group take Aerobics class  
**or**  
 A younger age group take Swimming class  
**or**  
 Similar

**Question 3**

**15 marks**

The arrows represent the different routes that a skier can take when skiing down a mountain. The circles on the diagram represent different points on the routes.

- (a) When leaving any particular point on the mountain a skier is equally likely to choose any of the available routes from that point. Fill in the boxes in the diagram which represent the probability that the skier will take that route.



- (b) (i) If the skier starts at point A, in how many different ways can the skier reach the point E?

<ol style="list-style-type: none"> <li>1. A → B → E</li> <li>2. A → B → D → E</li> <li>3. A → C → D → E</li> </ol>	$\left. \vphantom{\begin{matrix} 1. \\ 2. \\ 3. \end{matrix}} \right\}$	3 ways
--	---	--------

- (ii) If the skier starts at point A, find the probability that the skier will reach the point E.

<ol style="list-style-type: none"> <li>1. <math>\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}</math></li> <li>2. <math>\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}</math></li> <li>3. <math>\frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{12}</math></li> </ol>	$\left. \vphantom{\begin{matrix} 1. \\ 2. \\ 3. \end{matrix}} \right\}$	Probability = $\frac{1}{4} + \frac{1}{12} + \frac{1}{12}$ $= \frac{5}{12}$
--	---	---



**Question 4****10 marks**

A football strip consists of a shirt, shorts and socks.

Aspen United has two shirts, blue and green, from which to select. They also can select from three different colours of shorts and five different colours of socks, including red in each case.

- (a) Calculate how many different strips Aspen United can have.

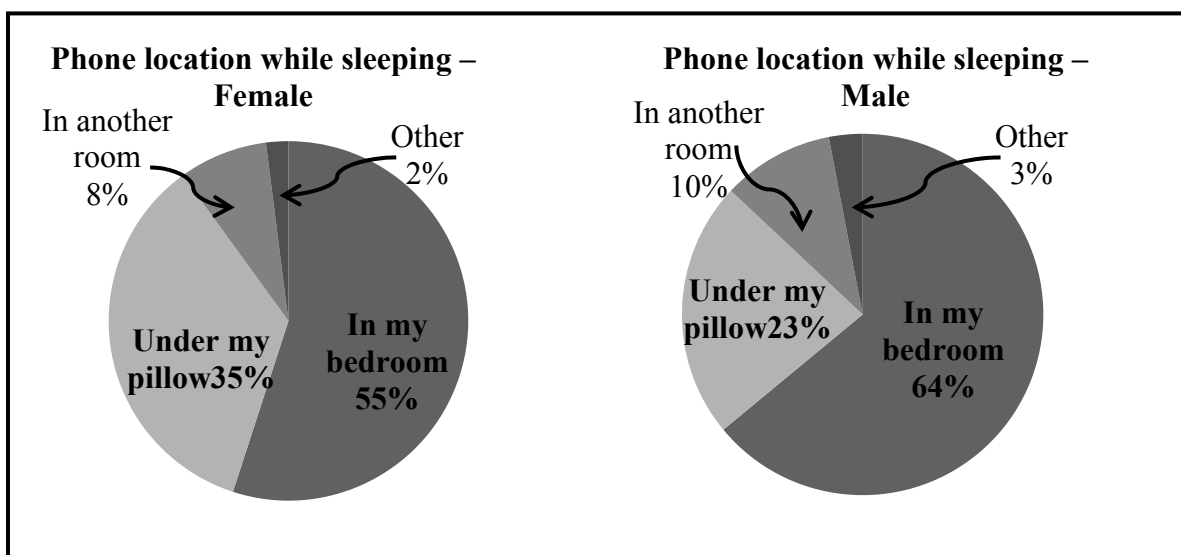
$$\text{Different strips: } 2 \times 3 \times 5 = 30$$

- (b) Willow Celtic plays in an all red strip. When Aspen United plays Willow Celtic, Aspen United are not allowed to use their red shorts or their red socks. Calculate how many different strips Aspen United can have when they play Willow Celtic.

$$\text{Different strips: } 2 \times 2 \times 4 = 16$$

**Question 5****25 marks**

In total 7150 second level school students from 216 schools completed the 2011/2012 phase 11 *CensusAtSchool* questionnaire. The questionnaire contained a question relating to where students keep their mobile phones while sleeping.



- (a) Given that this question was answered by 4171 girls and 2979 boys, calculate how many female students kept their mobile phones under their pillows.

$$\begin{aligned}
 \text{Girls – phone under pillow} &= 35\% \text{ of } 4171 \\
 &= 4171 \times 0.35 \\
 &= 1459.85 \\
 &= 1460 \quad (\text{or } 1459.85 \text{ or } 1459)
 \end{aligned}$$

- (b) Calculate the overall percentage of students who kept their mobile phones under their pillows.

$$\begin{aligned}
 \text{Total number of students} &= 7150 \\
 \text{Boys – phone under pillow} &= 23\% \text{ of } 2979 \\
 &= 685.17 \\
 &= 685 \quad (\text{or } 685.17 \text{ or } 686) \\
 \text{Total} &= 1460 + 685 = 2145 \quad (\text{or } 2145.02) \\
 \text{Percentage} &= \frac{2145}{7150} \times 100 \quad (\text{or } \frac{2145.02}{7150} \times 100) \\
 &= 30\% \quad (\text{or } 30.0002\%).
 \end{aligned}$$

- (c) A new pie chart is to be drawn showing the mobile phone location for all students. Calculate the measure of the angle that would represent the students who kept their mobile phones under their pillows.

$$\begin{aligned}
 \text{Angle} &= 30\% \text{ of } 360^\circ \\
 &= 360 \times 0.3 \\
 &= 108^\circ \quad (\text{or } 108.00072)^\circ
 \end{aligned}$$

**Question 6****30 marks**

The salaries, in €, of the different employees working in a call centre are listed below.

22000	16500	38000	26500	15000	21000	15500	46000
42000	9500	32000	27000	33000	36000	24000	37000
65000	37000	24500	23500	28000	52000	33000	25000
23000	16500	35000	25000	33000	20000	19500	16000

(a) Use this data to complete the grouped frequency table below.

Salary (€1000)	0 – 10	10 –20	20 –30	30–40	40 –50	50 –60	60 –70
No. of Employees	1	6	12	9	2	1	1

[Note: 10–20 means €10000 or more but less than €20000, etc.]

(b) Using mid-interval values find the mean salary of the employees.

<p>The mid - interval values are 5000, 15000, 25000, 35000, 45000, 55000, 65000</p> <p>Mean =</p> $\frac{(5000 \times 1) + (15\,000 \times 6) + (25\,000 \times 12) + (35\,000 \times 9) + (45\,000 \times 2) + (55\,000 \times 1) + (65\,000 \times 1)}{32}$ $= \frac{5000 + 90000 + 300000 + 315000 + 90000 + 55000 + 65000}{32}$ $= \frac{920000}{32}$ $= €28,750$
---

(c) (i) Outline another method which could have been used to calculate the mean salary.

Add up all the individual salaries and divide by 32.
--

(ii) Which method is more accurate? Explain your answer.

Answer	Adding up individual salaries and dividing by 32
--------	--

Reason	This gives the actual mean as estimates (mid-intervals) are not used.
--------	---

**Question 7****20 marks**

In a survey, 54 people were asked which political party they had voted for in the last three elections. The results are as follows:

30 had voted for the Conservatives

22 had voted for the Liberals

22 had voted for the Republicans

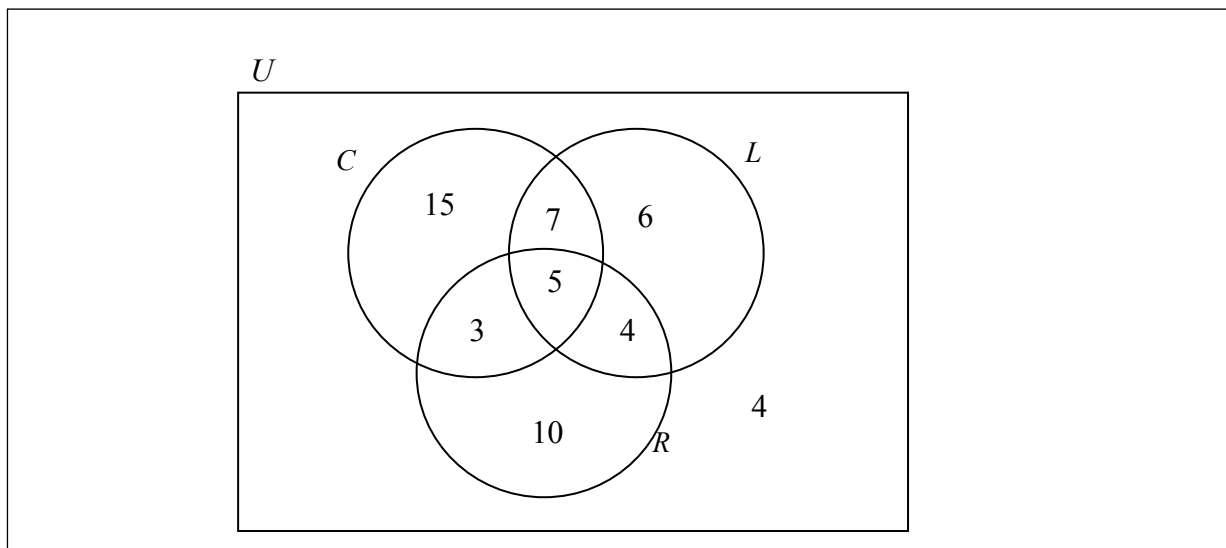
12 had voted for the Conservatives and for the Liberals

9 had voted for the Liberals and for the Republicans

8 had voted for the Conservatives and for the Republicans

5 had voted for all three parties.

- (a) Represent the information in a Venn diagram.



- (b) If one person is chosen at random, what is the probability that the person chosen did not vote in any of the three elections?

$$\text{Probability person did not vote} = \frac{4}{54} \text{ or } \frac{2}{27}$$

- (c) If one person is chosen at random, what is the probability that the person chosen voted for at least two different parties?

$$\text{Probability person voted for at least two parties} = \frac{3+5+7+4}{54} = \frac{19}{54}$$

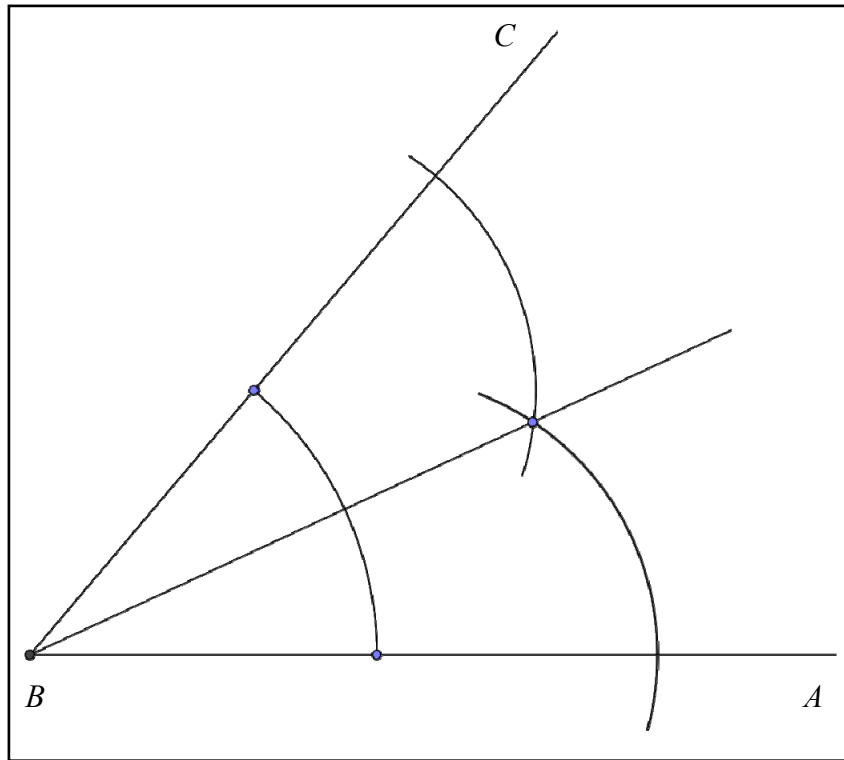
- (d) If one person is chosen at random, what is the probability that the person chosen voted for the same party in all three elections?

$$\text{Probability person voted for the same party} = \frac{15+6+10}{54} = \frac{31}{54}$$

**Question 8**

**10 marks**

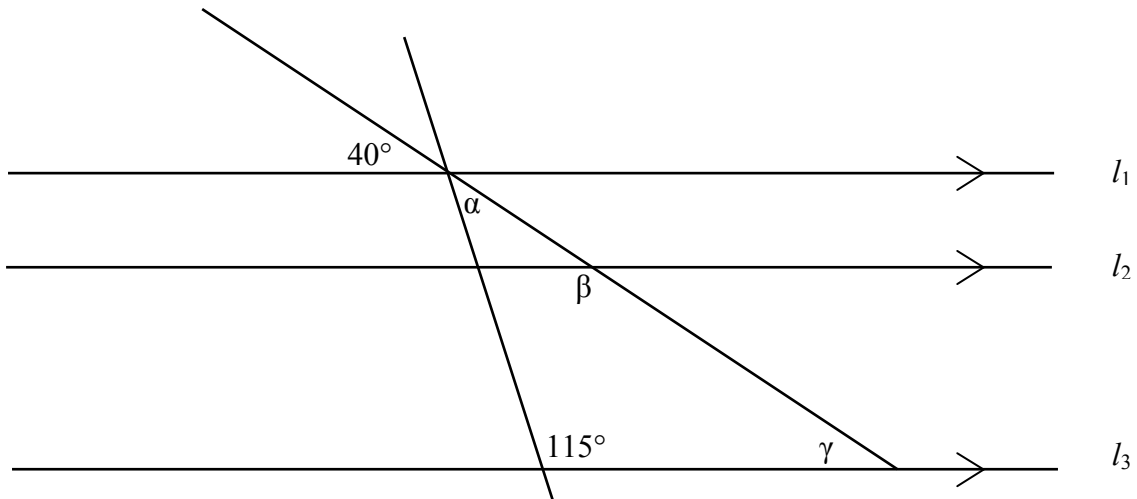
Construct the bisector of the  $\angle ABC$  below, using only a compass and straight edge. Show all construction work.



**Question 9**

**15 marks**

If  $l_1$ ,  $l_2$  and  $l_3$  are parallel lines, find the measure of the angles  $\alpha$ ,  $\beta$  and  $\gamma$ .



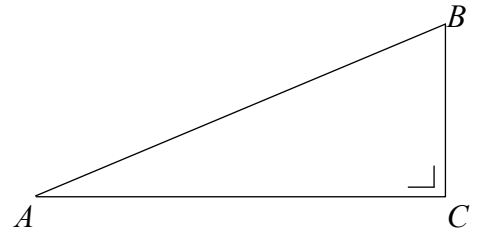
$\alpha$	=	$180 - (115 + 40)$	=	$25^\circ$
$\beta$	=	$180 - 40$	=	$140^\circ$
$\gamma$	=	$40^\circ$		

**Question 10****20 marks**

In the triangle  $ABC$ ,  $|AB| = 2$  and  $|BC| = 1$ .

(a) Find  $|AC|$ , giving your answer in surd form.

$$\begin{aligned} h^2 &= a^2 + b^2 \\ 2^2 &= |AC|^2 + 1^2 \\ \Rightarrow |AC| &= \sqrt{2^2 - 1^2} = \sqrt{3} \end{aligned}$$

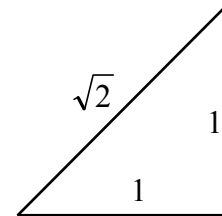


(b) Write  $\cos \angle BAC$  and hence find  $|\angle BAC|$ .

$$\begin{aligned} \cos \angle BAC &= \frac{\sqrt{3}}{2} \\ |\angle BAC| &= 30^\circ \end{aligned}$$

(c) Sketch a right angled isosceles triangle in which the equal sides are 1 unit each and use it to write  $\cos 45^\circ$  in surd form.

$$\begin{aligned} \text{Hypotenuse} &= \sqrt{1^2 + 1^2} = \sqrt{2} \\ \cos 45^\circ &= \frac{1}{\sqrt{2}} \end{aligned}$$



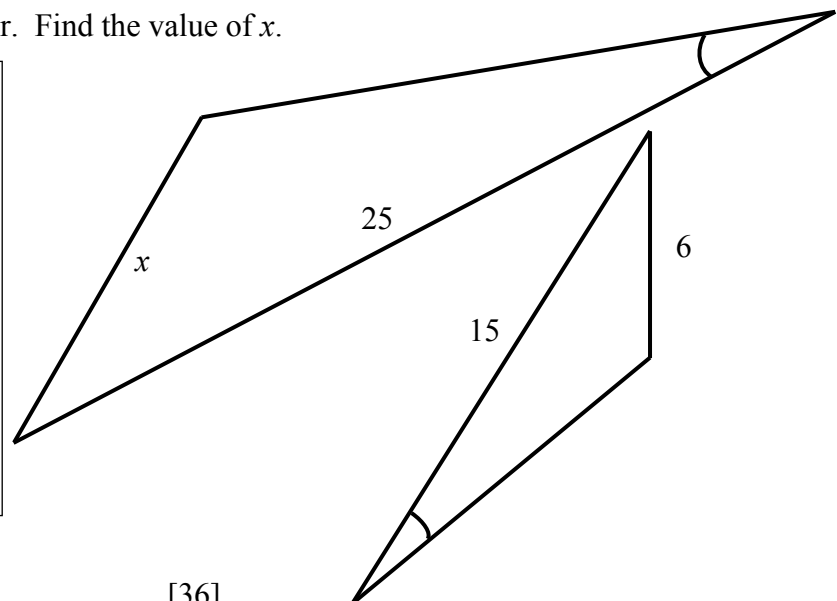
(d) Show that  $\cos 75^\circ \neq \cos 45^\circ + \cos 30^\circ$ .

$$\begin{aligned} \cos 75^\circ &= \frac{\sqrt{6} - \sqrt{2}}{4} = 0.2588 \\ \cos 45^\circ + \cos 30^\circ &= \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} = 0.7071 + 0.8660 = 1.5731 \\ (0.2588 &\neq 1.5731) \end{aligned}$$

**Question 11****10 marks**

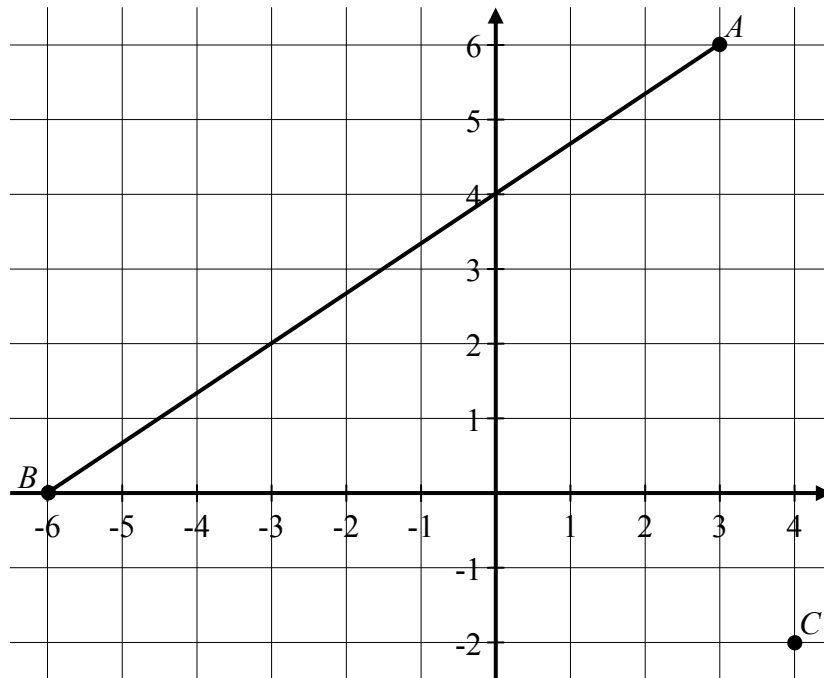
The two triangles shown are similar. Find the value of  $x$ .

$$\begin{aligned} \frac{x}{6} &= \frac{25}{15} \\ \Rightarrow x &= 10 \end{aligned}$$



Question 12

45 marks



- (a) Write the coordinates of  $A$ ,  $B$  and  $C$ .

$$A(3, 6) \quad B(-6, 0) \quad C(4, -2)$$

- (b) Find the co-ordinates of  $D$ , the mid-point of  $[AB]$ .

$$D = \left( \frac{3-6}{2}, \frac{6+0}{2} \right) = \left( -\frac{3}{2}, 3 \right)$$

- (c) Find the equation of the line  $AB$ .

$$\text{Slope } AB = \frac{0-6}{-6-3} = \frac{2}{3}$$

$$\text{Equation } AB: \quad y-0 = \frac{2}{3}(x+6) \quad \text{or} \quad y-6 = \frac{2}{3}(x-3)$$

or

$$y = \frac{2}{3}x + 4$$

$$2x - 3y + 12 = 0$$

- (d) Find the equation of the line through  $C$ , perpendicular to  $AB$ .

$$\text{Perpendicular slope} = -\frac{3}{2}$$

$$\begin{aligned} \text{Line through } C: \quad y + 2 &= -\frac{3}{2}(x - 4) \\ 3x + 2y - 8 &= 0 \end{aligned}$$

**or**

$$\begin{aligned} \text{The line is of the form } 3x + 2y + c &= 0 \\ (4, -2): 3(4) + 2(-2) + c &= 0 \Rightarrow c = -8 \\ 3x + 2y - 8 &= 0 \end{aligned}$$

- (e) Let  $E$  be the point where this perpendicular line through  $C$  intersects  $AB$ . Calculate the coordinates of the point  $E$ .

$$\begin{aligned} E \text{ the point of intersection of two lines} \quad 2x - 3y + 12 &= 0 \text{ (i)} \\ 3x + 2y - 8 &= 0 \text{ (ii)} \end{aligned}$$

$$\begin{aligned} 2 \times \text{(i)} \quad 4x - 6y &= -24 & \text{or} & \quad y = \frac{2x + 12}{3} \\ + 3 \times \text{(ii)} \quad 9x + 6y &= 24 & & \quad \Rightarrow 3x + 2\left(\frac{2x + 12}{3}\right) - 8 = 0 \\ & & & \quad \Rightarrow 9x + 4x + 24 - 24 = 0 \\ & \Rightarrow x = 0 & \text{and} & \quad y = 4 \end{aligned}$$

- (f) Which is the shorter distance,  $|CD|$  or  $|CE|$ ? Find this distance.

$$|CD| = \sqrt{\left(4 + \frac{3}{2}\right)^2 + (-2 - 3)^2} = \sqrt{55 \cdot 25} \quad \text{or} \quad 7 \cdot 433$$

$$|CE| = \sqrt{(4 - 0)^2 + (-2 - 4)^2} = \sqrt{52} \quad \text{or} \quad 7 \cdot 211$$

$|CE|$  is the shorter distance

**or**

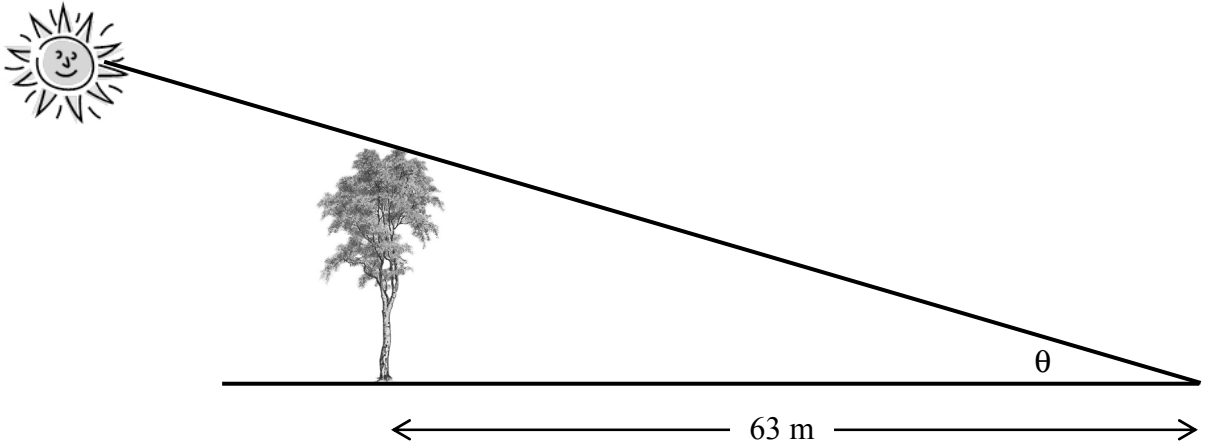
$|CE|$  (is the perpendicular distance and therefore is the shorter distance.)

$$|CE| = \sqrt{(4 - 0)^2 + (-2 - 4)^2} = \sqrt{52} \quad \text{or} \quad 7 \cdot 211$$



**Question 13****10 marks**

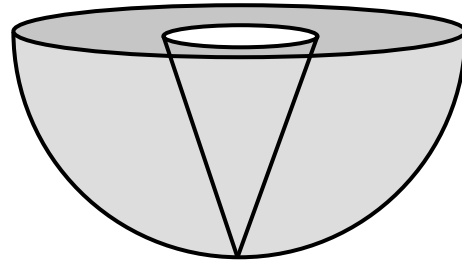
A tree 32 m high casts a shadow 63m long. Calculate  $\theta$ , the angle of elevation of the sun.  
Give your answer in degrees and minutes (correct to the nearest minute).



$\tan\theta$	$=$	$\frac{32}{63}$	<b>or</b>	$\tan\alpha$	$=$	$\frac{63}{32}$
$\Rightarrow\theta$	$=$	$26.9277$		$\Rightarrow\alpha$	$=$	$63.0723$
$\Rightarrow\theta$	$=$	$26^{\circ}55'39.64''$		$\Rightarrow\theta$	$=$	$90 - 63.0723$
	$=$	$26^{\circ}56'$			$=$	$26.9277$
					$=$	$26^{\circ}55'39.64''$
					$=$	$26^{\circ}56'$

**Question 14****20 marks**

A solid metal hemisphere has a radius of 12 cm.



(a) Calculate the volume of the hemisphere. Give your answer in terms of  $\pi$ .

Volume of hemisphere	$=$	$\frac{2}{3}\pi r^3$
	$=$	$\frac{2}{3}\times\pi\times 12^3$
	$=$	$1152\pi\text{ cm}^3$

- (b) A solid cone of radius 4 cm and height 12 cm is cut from the hemisphere. Calculate the volume of the cone. Give your answer in terms of  $\pi$ .

$$\begin{aligned}\text{Volume of cone} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\times\pi\times 4^2\times 12 \\ &= 64\pi\text{ cm}^3\end{aligned}$$

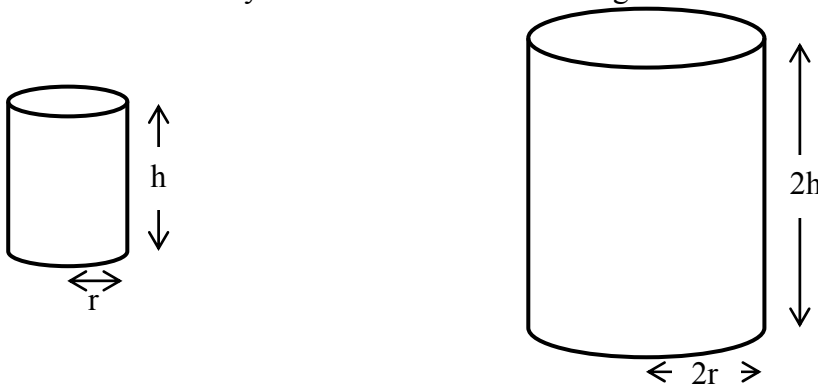
- (c) The remaining metal in the hemisphere is melted down and recast into cones of the same dimensions as the cone above. How many cones can be formed from the remaining metal?

$$\begin{aligned}\text{Remaining metal} &= 1152\pi - 64\pi \\ &= 1088\pi \\ \text{Volume of cone} &= 64\pi \\ \text{Number of cones} &= \frac{1088\pi}{64\pi} \\ &= 17 \\ &\text{or} \\ \text{Number of cones} &= \frac{1152\pi}{64\pi} - 1 \\ &= 18 - 1 \\ &= 17\end{aligned}$$

**Question 15**

**20 marks**

The dimensions of two solid cylinders are shown in the diagrams below.



- (a) Calculate the ratio of the curved surface area of the smaller cylinder to the curved surface area of the larger cylinder.

Curved surface area of a cylinder	=	$2\pi rh$
Curved surface area of small cylinder	=	$2 \times \pi \times r \times h$
	=	$2\pi rh$
Curved surface area of large cylinder	=	$2 \times \pi \times (2r) \times (2h)$
	=	$8\pi rh$
Ratio	=	$2\pi rh : 8\pi rh$
	=	$1 : 4$

- (b) Calculate the ratio of the volume of the smaller cylinder to the volume of the larger cylinder.

Volume of a cylinder	=	$\pi r^2 h$
Volume of small cylinder	=	$\pi \times r^2 \times h$
	=	$\pi r^2 h$
Volume of large cylinder	=	$\pi \times (2r)^2 \times (2h)$
	=	$8\pi r^2 h$
Ratio	=	$\pi r^2 h : 8\pi r^2 h$
	=	$1 : 8$

## Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C
No of categories	2	3	4
5 mark scale	0, 5	0, 4, 5	0, 2, 3, 5
10 mark scale	0, 10	0, 5, 10	0, 3, 7, 10
15 mark scale			0, 7, 12, 15

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

### Marking scales – level descriptors

#### *A-scales (two categories)*

- incorrect response (no credit)
- correct response (full credit)

#### *B-scales (three categories)*

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

#### *C-scales (four categories)*

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk.

Thus, for example, Scale 10C\* indicates that 9 marks may be awarded.

## Summary of mark allocations and scales to be applied

<b>Question 1 (20)</b> (a) 5C* (b) 5C* (c) 5C* (d) 5C	<b>Question 2 (30)</b> (a) 15C* (b) 5C (c) 5A (d) 5A	<b>Question 3 (15)</b> (a) 5C (b)(i) 5C (b)(ii) 5C	<b>Question 4 (10)</b> (a) 5A (b) 5A
<b>Question 5 (25)</b> (a) 5C (b) 15C (c) 5C*	<b>Question 6 (30)</b> (a) 15C (b) 10C (c) 5C	<b>Question 7 (20)</b> (a) 5C (b) 5A (c) 5B (d) 5B	<b>Question 8 (10)</b> 10A
<b>Question 9 (15)</b> 15C *	<b>Question 10 (20)</b> (a) 5C (b) 5C* (c) 5C (d) 5C	<b>Question 11 (10)</b> 10C	<b>Question 12 (45)</b> (a) 5C (b) 5C (c) 10C (d) 10C (e) 10C (f) 5C
<b>Question 13 (10)</b> 10C*	<b>Question 14 (20)</b> (a) 5C* (b) 5C* (c) 10C	<b>Question 15 (20)</b> (a) 10C (b) 10C	

## Detailed Marking Notes

The \* for units or rounding off is to be applied only if answer is fully correct. The \* is to be applied once only per question.

### Question 1

(a) Scale 5C\*

*High Partial Credit*

- $\sqrt{12^2 + 5^2}$

*Low Partial Credit*

- Some correct substitution into Pythagoras' theorem formula
- Correct formula from tables

Note: Apply \* if no units

(b) Scale 5C\*

*High Partial Credit*

- $\frac{1}{2}(5)(12)$

- $4 \times \frac{1}{2}(5)(12) = 120$

*Low Partial Credit*

- Area of a triangle =  $\frac{1}{2}(\text{base} \times \text{perpendicular height})$
- $2(5)(12)$
- Correct formula from tables

Note: Apply \* if no units

(c) Scale 5C\*

*High Partial Credit*

- $13^2$
- $17^2 - 4 \times 30$

*Low Partial Credit*

- $17^2$
- $4 \times 30$
- Area =  $l^2$  or  $l \times w$

Note: Accept candidate's answers from parts (a) and (b)

Note: Apply \* if no units

(d) Scale 5C

*High Partial Credit*

- $\frac{169}{0.4} = 422.5$
- $422.5 \times 0.2 = 84.5$
- $\frac{202.8}{0.4}$

*Low Partial Credit*

- $80 \times 50$
- 0.4 or 4000
- $0.4 \times 1.20$  or  $0.4 \times 0.2$
- $\frac{169}{0.4}$  or  $\frac{169}{4000}$  or  $422.5$
- Incorrect conversion of units
- Correct answer with no work

Note: Accept candidate's answers from previous parts

## Question 2

(a) Scale 15C\*

*High Partial Credit*

- 16 + ages placed correctly on both aerobics class and swimming class.

*Low Partial Credit*

- 1 to 15 ages placed correctly on either side

Note: It is not necessary to order the stem and leaf

Note: Apply \* if the key is missing or sides switched

(b) Scale 5C

*High Partial Credit*

- 37 to 39 **and** 29 to 31 given

*Low Partial Credit*

- One median only correct
- 37 or 39 or 29 or 31 given from a correctly ordered list

(c) Scale 5A

(d) Scale 5A

### Question 3

(a) Scale 5C

*High Partial Credit*

- 6, 7 or 8 probabilities correct

*Low Partial Credit*

- 1 to 5 probabilities correct

(b)(i) Scale 5C

*High Partial Credit*

- 2 paths correctly identified

*Low Partial Credit*

- 1 path correctly identified
- More than 3 paths listed with at least 1 correct

(b) (ii) Scale 5C

*High Partial Credit*

- The three probabilities  $\frac{1}{4}$ ,  $\frac{1}{12}$ ,  $\frac{1}{12}$  given

- Probability =  $\frac{1}{4} + \frac{1}{12} + \frac{1}{12}$

- Error in one probability with addition correctly completed

*Low Partial Credit*

- Probability for 1 or 2 paths correct

### Question 4

(a) Scale 5A

(b) Scale 5A



### Question 5

(a) Scale 5C

*High Partial Credit*

- $4171 \times 0.35$

*Low Partial Credit*

- 35%
- 0.35

(b) Scale 15C

*High Partial Credit*

- $\frac{2145}{7150} \times 100$        $\left( \text{or } \frac{2145 \cdot 02}{7150} \times 100 \right)$

*Low Partial Credit*

- 685      (or 685.17 or 686)

Note: Accept candidate's answer from part (a)

(c) Scale 5C\*

*High Partial Credit*

- $360 \times 0.3$
- 1 student =  $\frac{360}{7150} = 0.05^\circ$

*Low Partial Credit*

- 30% of 360
- Indication that the answer is a fraction of 360
- Identifies 7150 as  $360^\circ$

Note: Accept candidate's answer from part (b)

Note: Apply \* if units missing

### Question 6

(a) Scale 15C

*High Partial Credit*

- Entry  $\pm 2$  in not more than 2 frequencies.

*Low Partial Credit*

- Some correct entry in table

(b) Scale 10C

*High Partial Credit*

- Answer = 28.75

*Low Partial Credit*

- Error in numerator - e.g. incorrect mid-interval values or no mid-interval values
- Error in denominator
- Effort at mid-interval values
- Sum of frequencies indicated

(c) Scale 5C

*High Partial Credit*

- Correct method and correct answer. No reason given.

*Low partial Credit*

- Correct method only.

### Question 7

(a) Scale 5C

*High Partial Credit*

- 2 – 7 correct entries.

*Low Partial Credit*

- 1 correct entry

(b) Scale 5A

Note: Accept value from candidate's Venn diagram except where negative values used

Note: Answer must be consistent with candidate's Venn diagram

(c) Scale 5B

*Partial Credit*

- $\frac{3+5+7+4}{54}$
- Not more than one element incorrect or omitted

Note: Accept values from candidate's Venn diagram unless probability  $> 1$

Note: Answer must be consistent with candidate's Venn diagram

(d) Scale 5B

*Partial Credit*

- $\frac{15+6+10}{54}$
- Not more than one element incorrect or omitted

Note: Accept values from candidate's Venn diagram unless probability  $> 1$

Note: Accept value from candidate's Venn diagram except where negative values used

Note: Answer must be consistent with candidate's Venn diagram

### Question 8

Scale 10A

Note: Accept tolerance of  $\pm 2^\circ$

Note: All constructions must be shown

### Question 9

Scale 15C\*

*High Partial Credit*

- 2 angles correct

*Low Partial Credit*

- 1 angle correct
- Some correct work on alternate / corresponding/vertically opposite angles
- Some correct work on the angles in a straight line adding to  $180^\circ$  or  $\beta + \chi = 180^\circ$
- Some correct work on the angles of a triangle adding to  $180^\circ$

Note: Apply \* if units missing

### Question 10

(a) Scale 5C

*High Partial Credit*

- $\sqrt{2^2 - 1^2}$
- Answer not in surd form

*Low Partial Credit*

- Some correct substitution into Pythagoras' theorem formula
- $|\angle BAC| = 30^\circ$  or  $|\angle ABC| = 60^\circ$
- Correct formula from tables

(b) Scale 5C\*

*High Partial Credit*

- $\cos \angle BAC = \frac{\sqrt{3}}{2}$

Note: Accept candidate's answer from part (a) if cos ratio  $< 1$

Note: Apply \* if units missing

*Low Partial Credit*

- Some correct use of cos ratio
- Indication that  $\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$
- $|\angle BAC|$  found without use of cos ratio

Note: Accept candidate's answer from part (a) if cos ratio  $\geq 1$

(c) Scale 5C

*High Partial Credit*

- Sketch drawn and  $\sqrt{2}$  found or sketch drawn and  $\sqrt{2}$  shown on diagram
- Answer not in surd form

*Low Partial Credit*

- Correct sketch drawn
- $\cos 45^\circ = \frac{1}{\sqrt{2}}$  or  $\frac{\sqrt{2}}{2}$  from tables/calculator
- Correct sketch drawn and  $\cos 45^\circ = 0.7071$  from calculator
- Indication that  $\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$
- Some correct substitution into Pythagoras' theorem formula

(d) Scale 5C

*High Partial Credit*

- Both correct but not in forms in which they can be easily compared
- Calculator in incorrect mode

*Low Partial Credit*

- $\cos 75^\circ$  found correctly
- $\cos 45^\circ$  and/ or  $\cos 30^\circ$  found or transferred from part (b) and part(c)

### Question 11

Scale 10C

*High Partial Credit*

- $\frac{x}{6} = \frac{25}{15}$  or equivalent

*Low Partial Credit*

- Corresponding sides/angles identified
- One correct relevant ratio
- Indication that corresponding sides are proportional

### Question 12

(a) Scale 5C

*High Partial Credit*

- 2 points correct

*Low Partial Credit*

- 1 point correct
- All 3 reversed ( $y, x$ )

(b) Scale 5C

*High Partial Credit*

- Correct substitution into midpoint formula
- Both  $x$  and  $y$  reversed in substitution

*Low Partial Credit*

- Some correct substitution into midpoint formula
- One ordinate correct
- Coordinates reversed
- Correct formula from tables

Note: Correct answer written in part (b) merits full marks.

Note: Accept candidate's points from part (a)

(c) Scale 10C

*High Partial Credit*

- Error in slope but continues correctly
- Correct slope with 1 incorrect substitution into a line formula
- Correct slope but both  $x$  and  $y$  reversed in substitution

*Low Partial Credit*

- Some correct substitution into slope formula
- Slope found
- Some correct substitution into line formula
- Indication that slope =  $\frac{\text{rise}}{\text{run}}$
- Correct formula from tables

Note: It is not necessary to write the equation in the form  $ax + by + c = 0$  for full marks.

(d) Scale 10C

*High Partial Credit*

- Error in slope but continues correctly
- Correct slope with 1 incorrect substitution into a line formula
- Correct slope but both  $x$  and  $y$  reversed in substitution

*Low Partial Credit*

- Some correct substitution into line formula
- Slope correct
- Indication that the product of the slopes of perpendicular lines is  $-1$
- $3x + 2y + c = 0$
- Correct formula from tables

Note: It is not necessary to write the equation in the form  $ax + by + c = 0$  for full marks

(e) Scale 10C

*High Partial Credit*

- One (correct) ordinate only found
- 1 error in simplifying equations in parts (c) or (d) but solved correctly

*Low Partial Credit*

- More than 1 error in simplifying equations from parts (c) and (d) in this part and continues with some correct work
- Equations from parts (c) **and** (d) correctly simplified in this part
- Point  $E$  correctly read from graph. Graph must have perpendicular line correctly drawn from  $C$  - otherwise 0 marks
- Equations from parts (c) and (d) correctly simplified but continues incorrectly

Note: Accept candidate's equations from parts (c) and (d)

(f) Scale 5C

*High Partial Credit*

- $|CD|$  and  $|CE|$  both found but no indication of shorter distance

*Low Partial Credit*

- Some correct substitution into distance formula
- $|CD|$  only found
- States  $|CE|$  ( or perpendicular distance) is the shorter but  $|CE|$  not found
- $|CE|$  found but not identified
- Correct formula from tables

### Question 13

Scale 10C\*

*High Partial Credit*

- $\theta = 26.9277$
- Calculator in incorrect mode

*Low Partial Credit*

- $\tan\theta = \frac{32}{63}$
- Correct relevant trigonometric formula
- Indication that  $\tan = \frac{\text{opposite}}{\text{adjacent}}$

Note: Apply \* if answer not correctly rounded to the nearest minute and/or no units

### Question 14

(a) Scale 5C\*

*High Partial Credit*

- Volume of sphere found
- $\frac{2}{3} \times \pi \times 1728$

*Low Partial Credit*

- Correct or some correct substitution into volume of sphere/hemisphere formula
- Volume of hemisphere =  $\frac{1}{2}$  (Volume of sphere)
- Incorrect relevant formula used
- Correct formula from tables

Note: Apply \* if answer not written in terms of  $\pi$  and/or no units

(b) Scale 5C\*

*High Partial Credit*

- $\frac{1}{3} \times \pi \times 16 \times 12$

*Low Partial Credit*

- Correct or some correct substitution into volume of cone formula
- Incorrect relevant formula used
- Correct formula from tables

Note: Apply \* if answer not written in terms of  $\pi$  and/or no units (if not applied in part (a))

(c) Scale 10C

*High Partial Credit*

- $\frac{1152\pi}{64\pi} = 18$
- $\frac{1088\pi}{64\pi}$

*Low Partial Credit*

- Subtraction of candidate's volumes
- Indication of division by  $64\pi$

Note: Accept candidate's values from previous parts provided answer in part (a) is greater than answer in part (b) – otherwise 0 marks

## Question 15

(a) Scale 10C

*High Partial Credit*

- Both  $2\pi rh$  and  $8\pi rh$  written
- Values substituted for r and h and correct ratio found

*Low Partial Credit*

- Some correct substitution into curved surface area of cylinder formula
- $2 \times \pi \times r \times h$  and /or  $2 \times \pi \times 2r \times 2 h$  written
- Incorrect relevant formula used
- Values substituted for r and h and ratio incorrect
- Correct answer without work
- Correct formula from tables



(b) Scale 10C

*High Partial Credit*

- Both  $\pi r^2 h$  and  $8\pi r^2 h$  written
- Values substituted for r and h and correct ratio found

*Low Partial Credit*

- Some correct substitution into volume of cylinder formula
- $\pi \times r^2 \times h$  and/or  $\pi \times (2r)^2 \times 2h$  written
- Values substituted for r and h and ratio incorrect
- Incorrect relevant formula used
- Correct answer without work
- Correct formula from tables

## BONUS MARKS FOR ANSWERING THROUGH IRISH

Bonus marks are applied separately to each paper as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded **down**.  
(e.g. 198 marks  $\times$  5% = 9.9  $\Rightarrow$  bonus = 9 marks.)

If the mark awarded is above 225, the following table applies:

Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)	Bunmharc (Marks obtained)	Marc Bónais (Bonus Mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0



