



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Junior Certificate Examination 2014

Mathematics (Project Maths – Phase 3)

Paper 1

Higher Level

Friday 6 June – Afternoon, 2:00 to 4:30

300 marks

Examination number

Centre stamp

Running total	
---------------	--

For examiner			
Question	Mark	Question	Mark
1		11	
2		12	
3		13	
4		14	
5			
6			
7			
8			
9			
10		Total	

Grade

Instructions

There are 14 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Question 14 carries a total of 50 marks.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 2

(Suggested maximum time: 5 minutes)

- (a) John thinks that he has a method for finding **all** prime numbers. He says that if he uses the formulas in the table below, he will generate the prime numbers. He also says that these formulas will generate **only** the prime numbers.

(i) Complete the table.

p	$6p + 1$	$6p + 5$
0	1	5
1		
2		
3		
4		
5		

(ii) Give two reasons why his method is not fully correct.

Reason 1:

Reason 2:

- (b) The Swiss mathematician and physicist, Euler, first noticed (in 1772) that the expression $n^2 - n + 41$ gives a prime number for all positive integer values of n less than 41.

Explain why it does not give a prime number for $n = 41$.

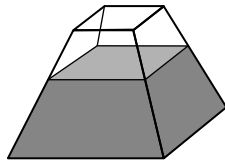


Question 6

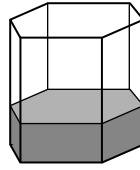
(Suggested maximum time: 10 minutes)

Below are three containers, labelled 1, 2, and 3.

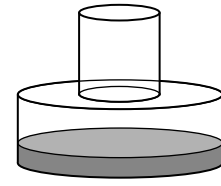
Water is poured into each container at a constant rate, until it is full.



1

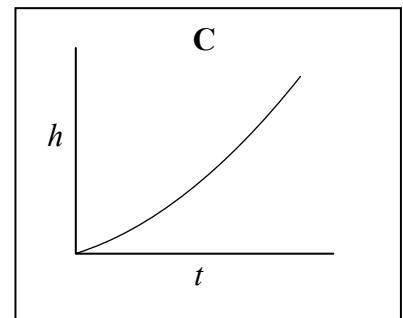
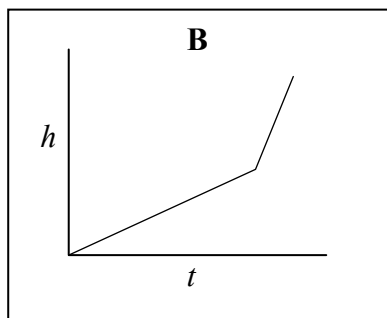
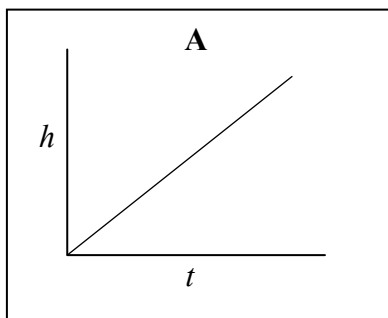


2



3

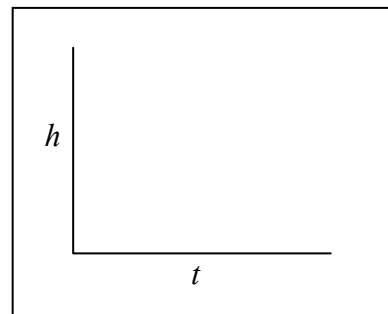
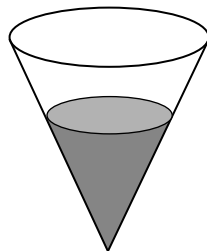
The three graphs, **A**, **B**, and **C**, show the height of the water, h , in the containers after time t .



(a) Write **A**, **B**, and **C** in the table below to match each container to its corresponding graph.

Container	1	2	3
Graph			

(b) Another container is shown below. Water is also poured into this container at a constant rate until it is full. Sketch the graph you would expect to get when plotting height (h) against time (t) for this container.



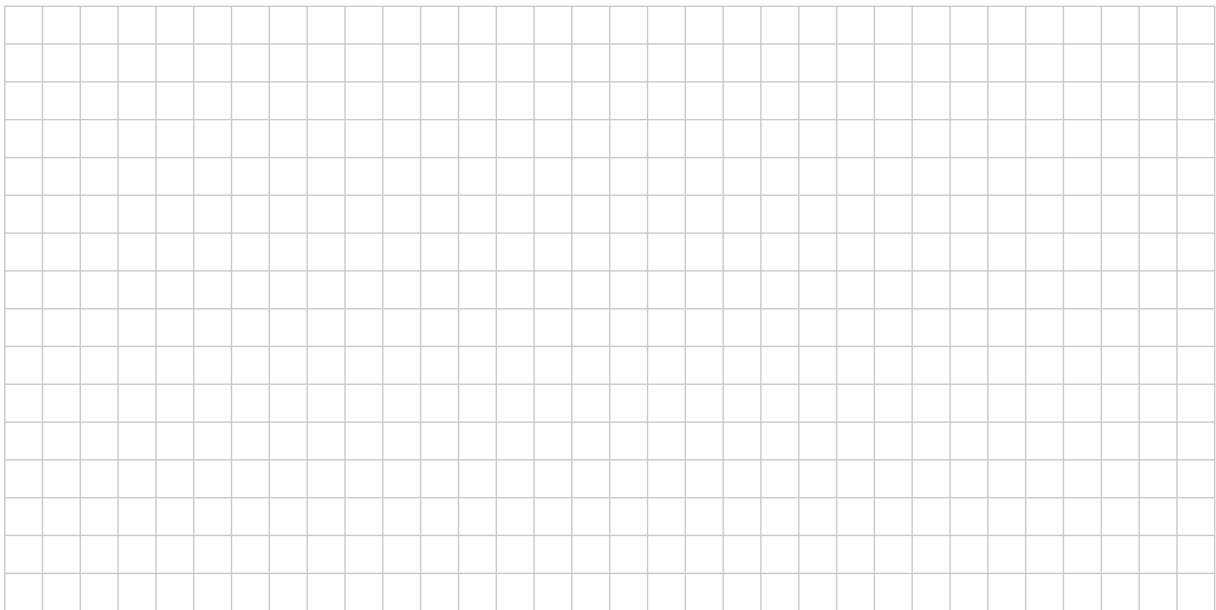
Question 7

(Suggested maximum time: 10 minutes)

Last year Elena had a gross income of €36 960.
She had to pay Universal Social Charge (USC) and income tax on her gross income.
The rates and bands of USC are as follows.

Income band	Rate of USC
Up to €10 036	2%
Between €10 036 and €16 016	4%
Above €16 016	7%

- (i) Find the amount of USC that was deducted from Elena’s gross income last year.



- (ii) The standard rate of income tax was 20% and the higher rate was 41%.
The standard rate cut-off point was €32 800.
Elena paid a total of €4965·60 income tax last year.

Find Elena’s tax credits for the year.



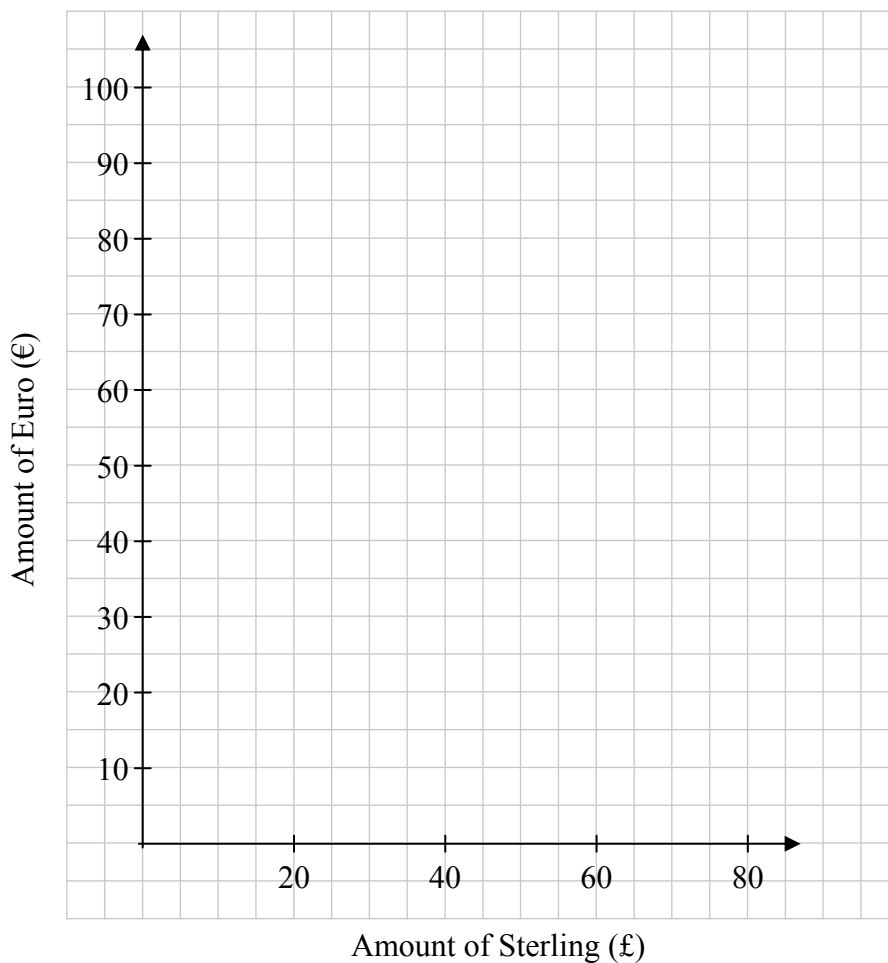
Question 9**(Suggested maximum time: 15 minutes)**

Jack and Sarah are going on a school tour to England. They investigate how much different amounts of sterling (£) will cost them in euro (€). They each go to a different bank.

Their results are shown in the table below.

Amount of sterling (£)	Cost in euro (€) for Jack	Cost in euro (€) for Sarah
20	33	24
40	56	48
60	79	72
80	102	96

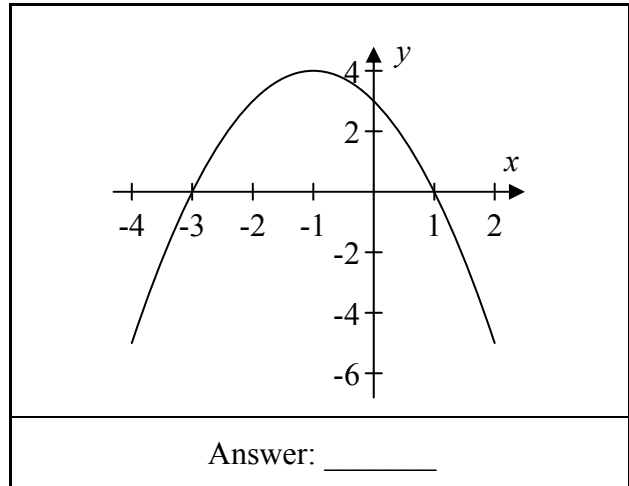
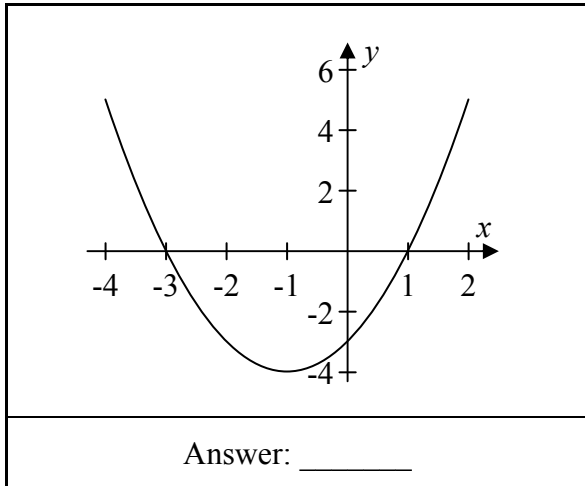
- (i) On the grid below, draw graphs to show how much the sterling will cost Jack and Sarah, for up to £80.



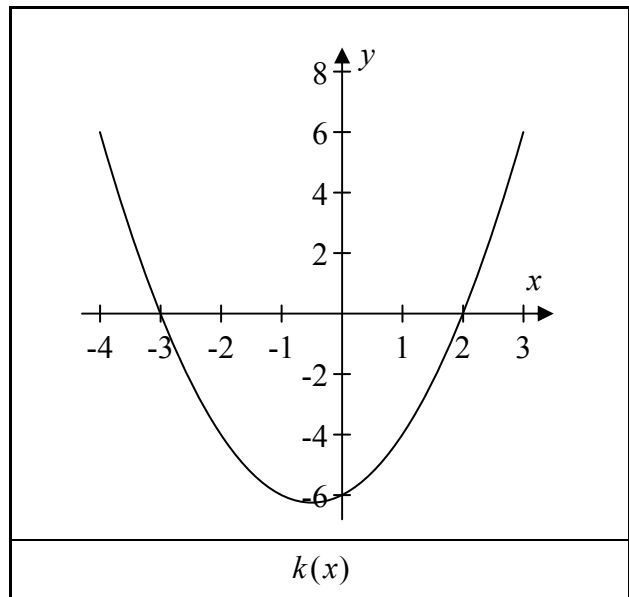
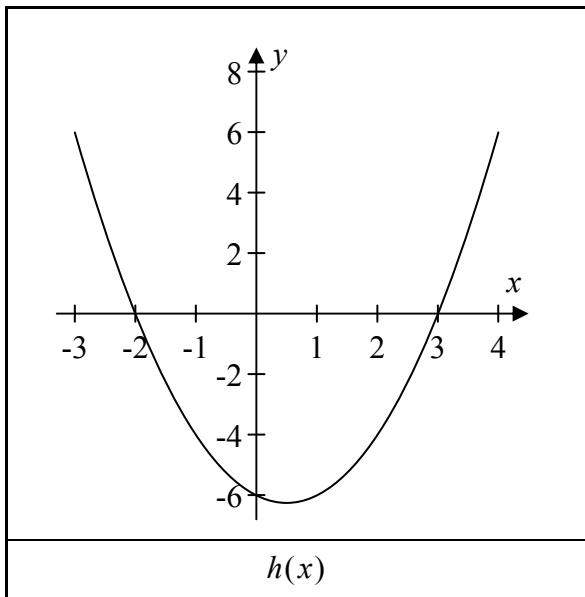
Question 10

(Suggested maximum time: 10 minutes)

- (a) The graphs of the functions $f(x) = x^2 + 2x - 3$ and $g(x) = -x^2 - 2x + 3$ are shown below. Identify each graph by writing $f(x)$ or $g(x)$ in the space provided below the graph.



- (b) The graphs of the functions $y = h(x)$ and $y = k(x)$ are shown below.



Write down the roots of each function.
Hence, or otherwise, write down an equation for each function.

Roots of $h(x)$: Equation: $h(x) =$
Roots of $k(x)$: Equation: $k(x) =$

(b) Simplify $(3x^3 - 2x^2 - 3x + 2) \div (x - 1)$.



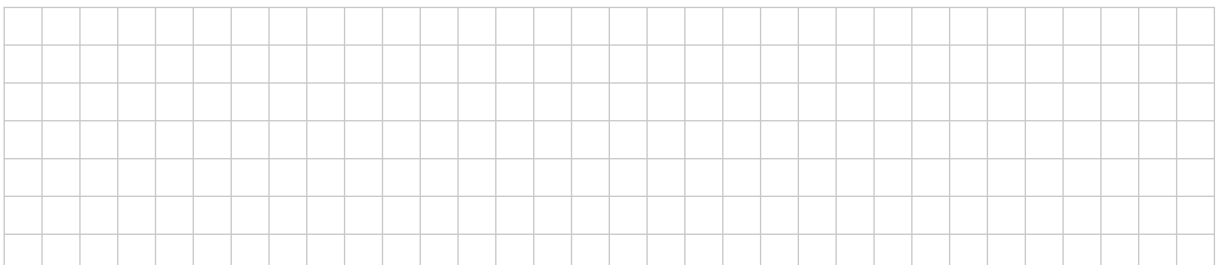
(c) (i) Solve the simultaneous equations:

$$2x - 3y = 18$$

$$5x + 9y = -10.$$



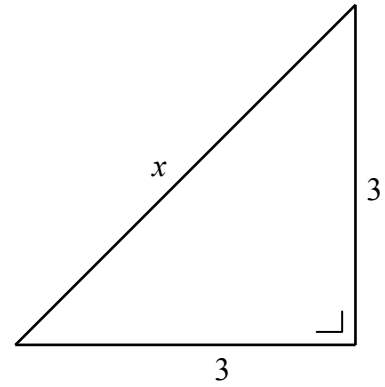
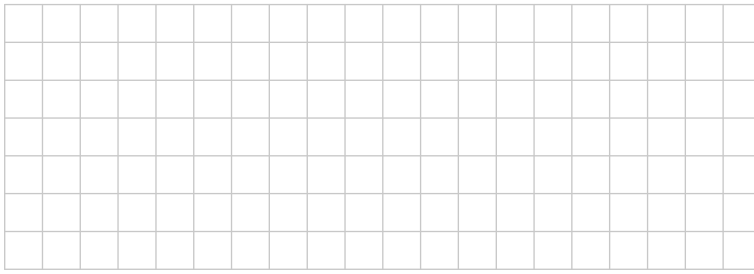
(ii) Verify your answer to (c)(i).



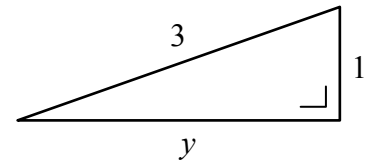
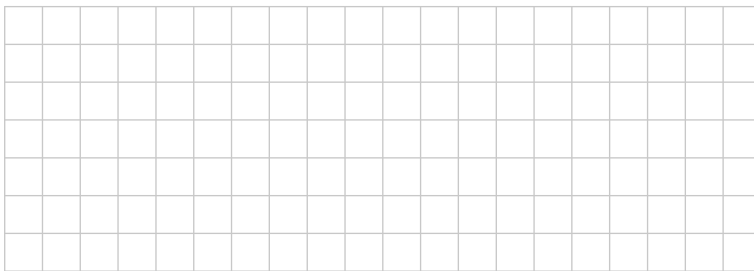
Question 13

(Suggested maximum time: 5 minutes)

- (i) Use the diagram on the right to calculate the value of x .
Give your answer in surd form.

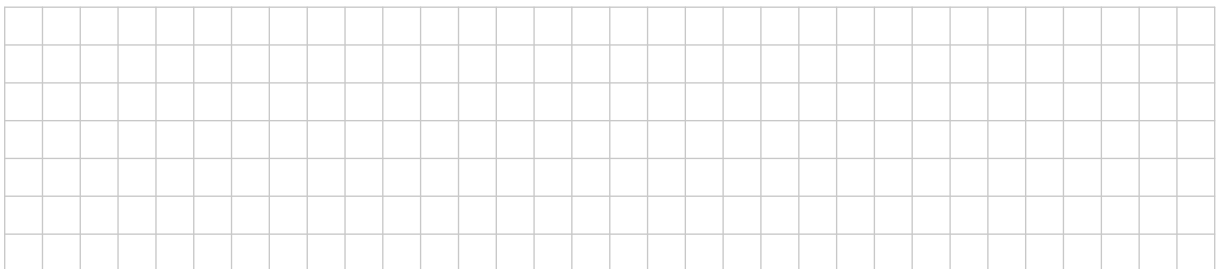
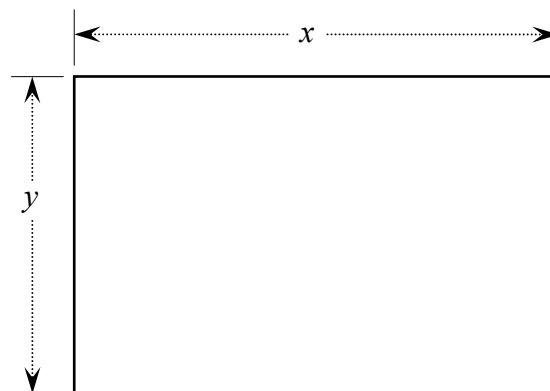


- (ii) Use the diagram below to calculate the value of y . Give your answer in surd form.



- (iii) A rectangle with sides of length x and y is drawn using the values of x and y from parts (i) and (ii), as shown below.

Write the **perimeter** of this rectangle in the form $a\sqrt{2}$, where $a \in \mathbb{N}$.



Page	running
------	---------

Question 14

(Suggested maximum time: 20 minutes)

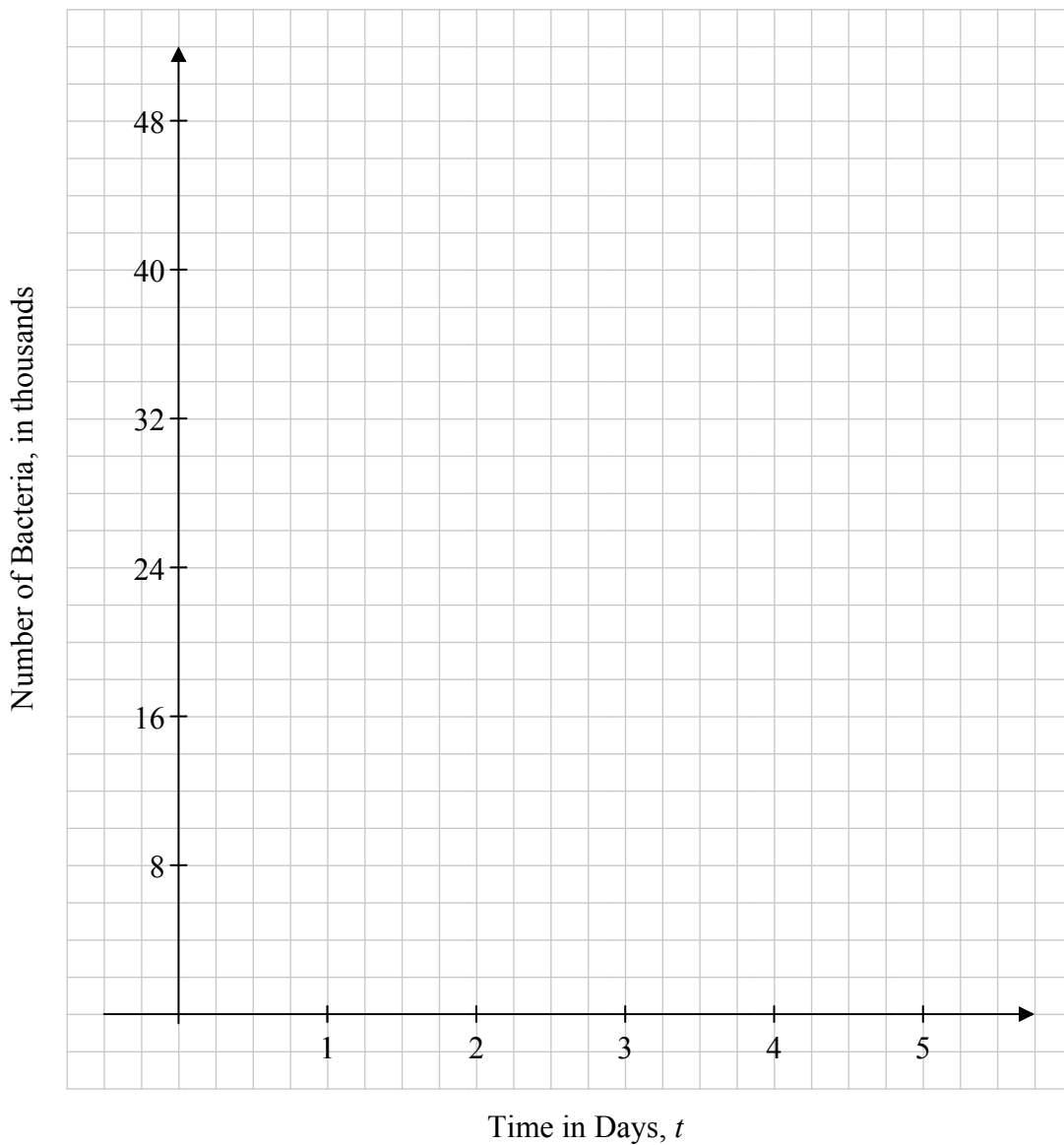
Paul and Marie have been studying the growth of a particular bacterium in school. They each come up with a function to predict the number of bacteria in a colony, in thousands, after t days. They both assume that there are 1000 bacteria in the colony at the beginning ($t=0$).

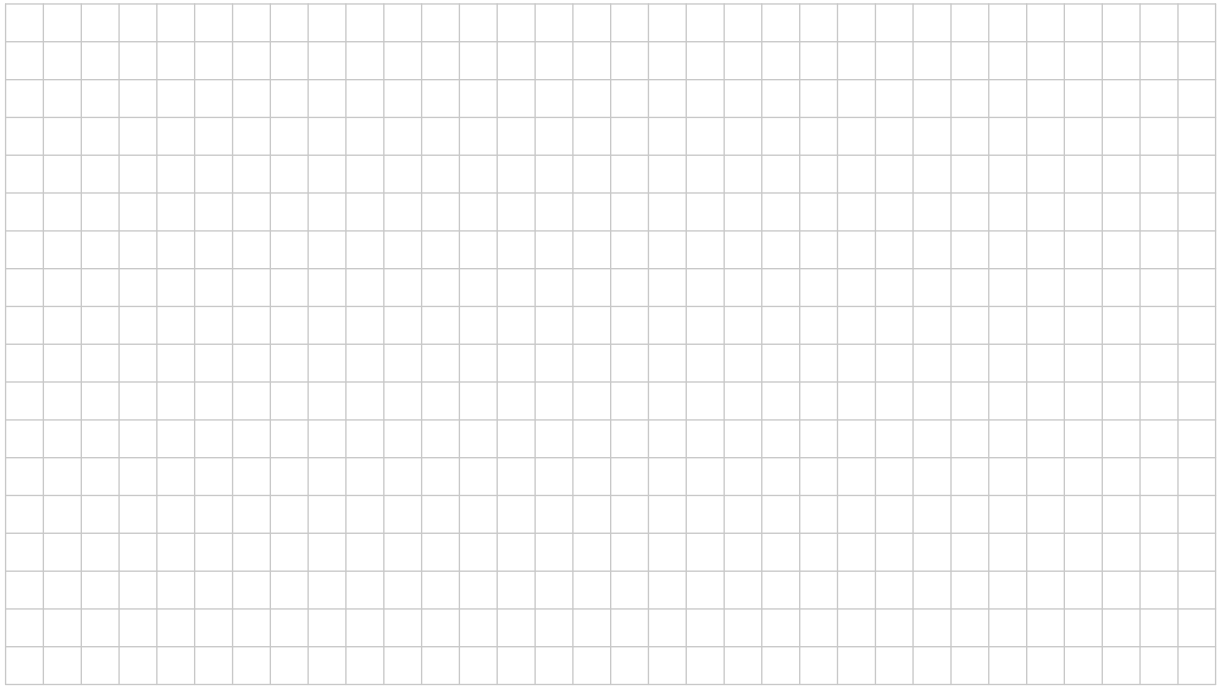
Paul comes up with the function: $f : t \mapsto 2^t$.

Marie comes up with the function: $g : t \mapsto t^2 + 2t + 1$.

- (i) On the grid below, draw the graphs of $y = f(t)$ and $y = g(t)$ in the domain $0 \leq t \leq 5$, $t \in \mathbb{R}$.

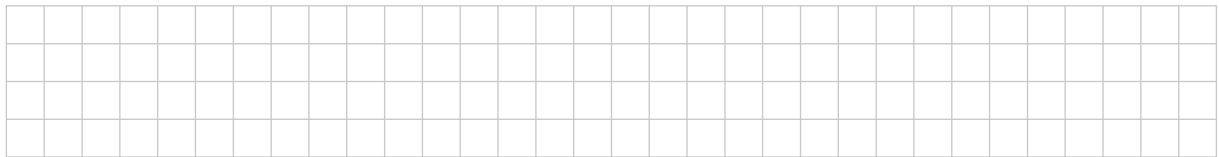
There is room for working out on the next page.



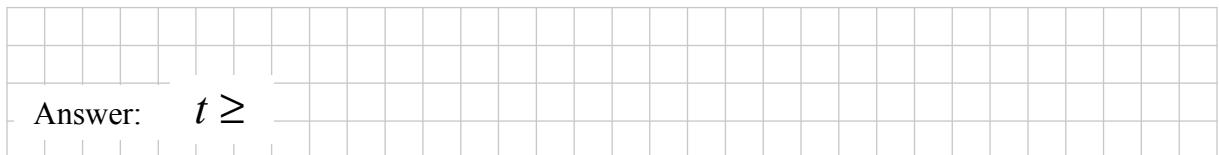


For parts (ii), (iii), and (iv), you **must** show your working out on the diagram on the previous page.

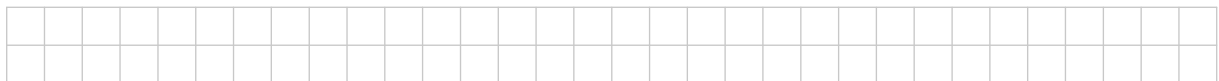
- (ii) Use your graphs to find the difference in the number of bacteria predicted by Paul and the number of bacteria predicted by Marie after 2.5 days.



- (iii) Use your graphs to estimate the range of values of t for which **both** Paul and Marie predict that there will be at least 20 000 bacteria in the colony.



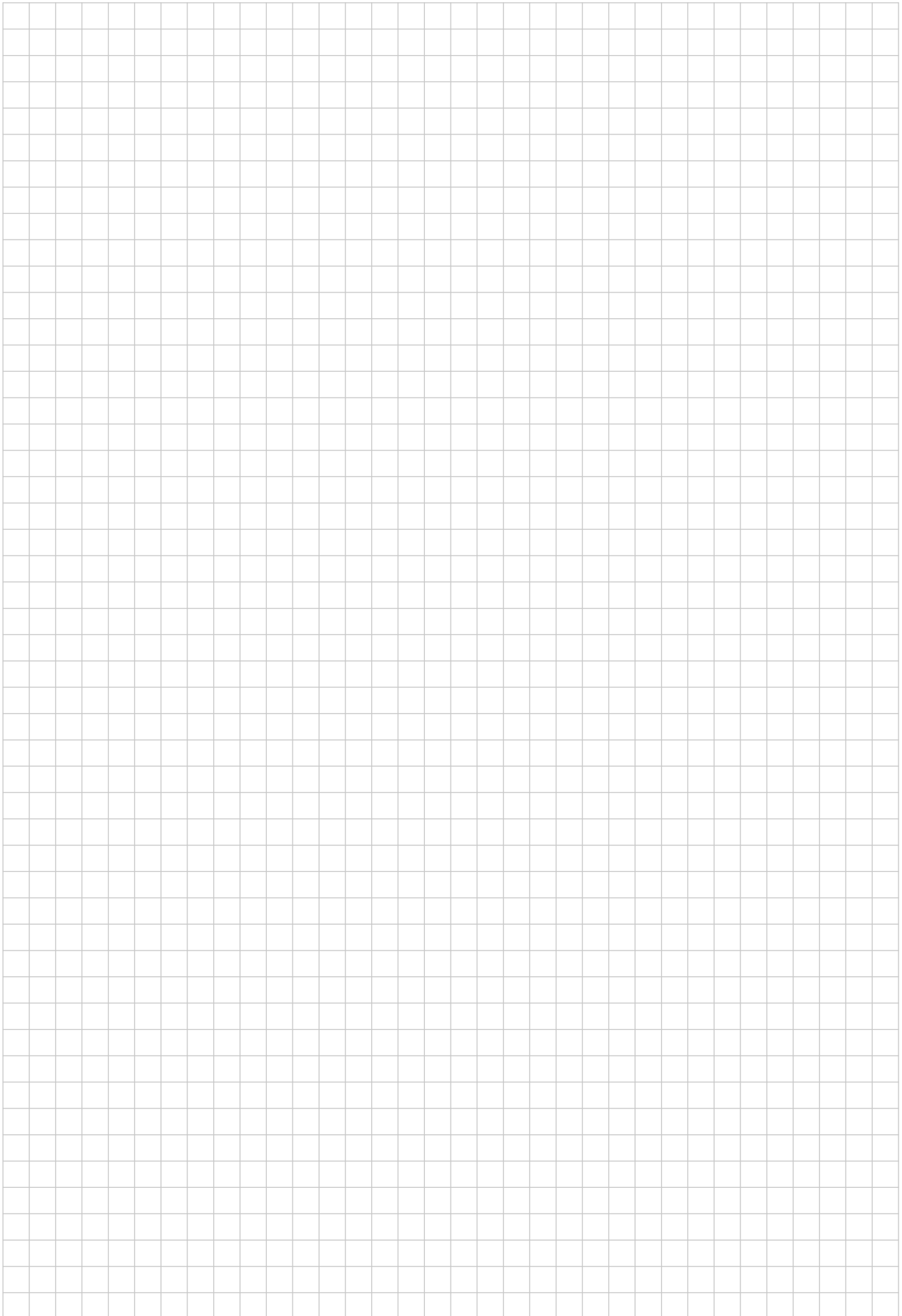
- (iv) By extending your graphs, estimate the value of t (other than $t = 0$) for which the number of bacteria predicted by Paul and the number of bacteria predicted by Marie will be the same.



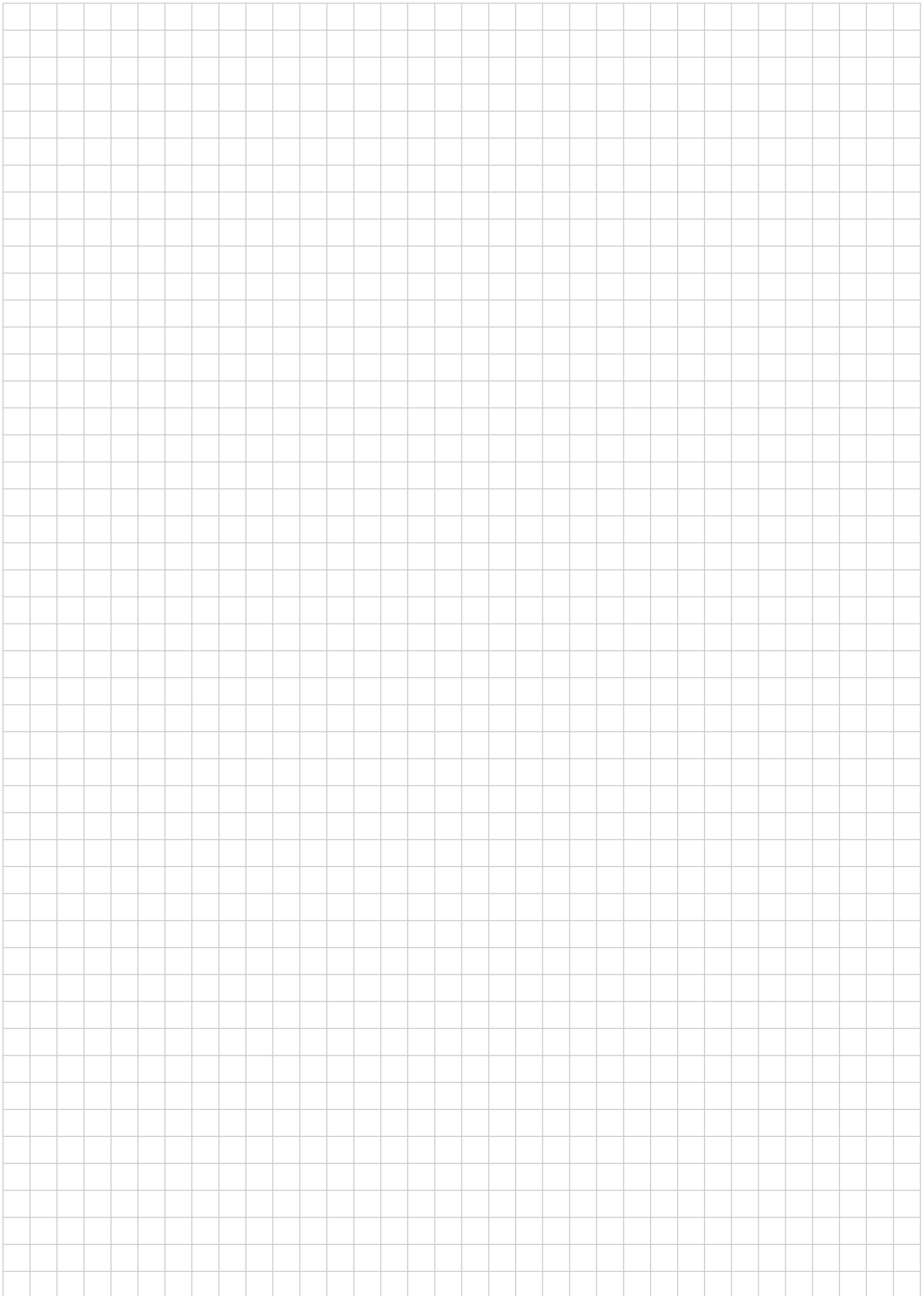
- (v) The actual number of bacteria after two weeks (14 days) is roughly 1.6×10^7 . Based on this, which formula would you say gives the better prediction for the number of bacteria? Explain your answer.



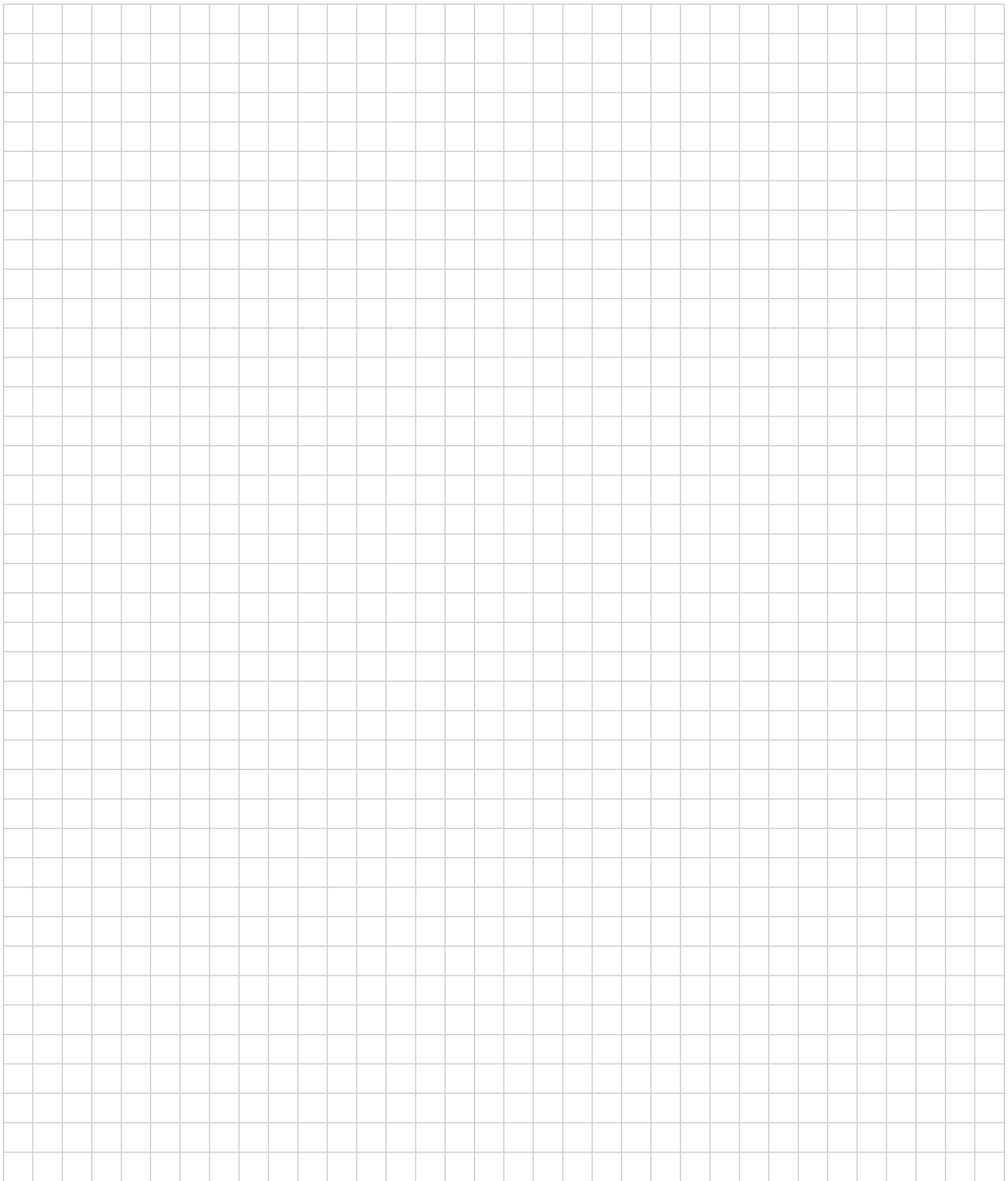
You may use this page for extra work.



You may use this page for extra work.



Page	running
------	---------



Junior Certificate 2014 – Higher Level

Mathematics (Project Maths – Phase 3) – Paper 1

Friday 6 June
Afternoon, 2:00 to 4:30