



**Coimisiún na Scrúduithe Stáit**  
State Examinations Commission

**Junior Certificate 2015**

**Marking Scheme**

**Mathematics**

**Ordinary Level**

## **Note to teachers and students on the use of published marking schemes**

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

## **Future Marking Schemes**

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Junior Certificate Examination 2015

# Mathematics

Paper 1

Ordinary Level

## Model Solutions – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

## **Instructions**

There are 12 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

**You will lose marks if all necessary work is not clearly shown.**

**You may lose marks if the appropriate units of measurement are not included, where relevant.**

**You may lose marks if your answers are not given in simplest form, where relevant.**

Write the make and model of your calculator(s) here:

**Question 1****25 Marks**

(a) Find the value of each of the following.

(i)  $2 \cdot 5 - 1 \cdot 5 \times 0 \cdot 1$

$$2 \cdot 5 - 0 \cdot 15 = 2 \cdot 35.$$

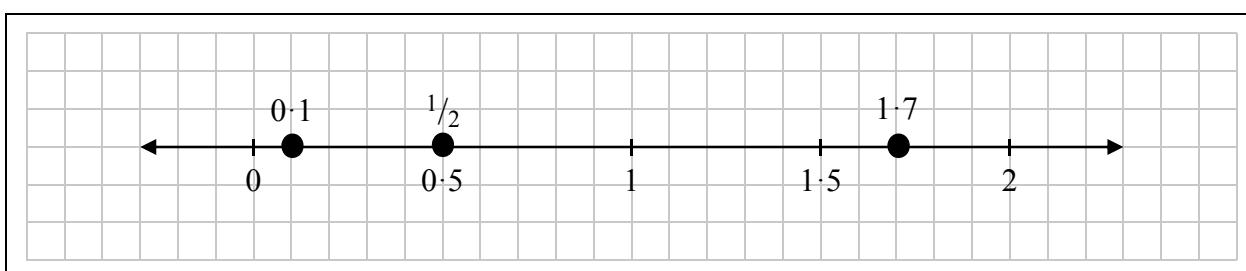
(ii)  $\sqrt{5+1 \cdot 25}$

$$\sqrt{6 \cdot 25} = 2 \cdot 5.$$

(iii)  $(-2)^3$

$$-8.$$

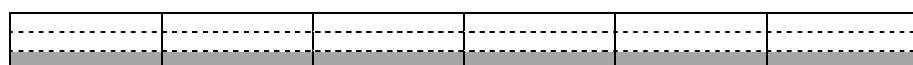
(b) Show each of the following numbers on the number line below. Label each one clearly.



(c) (i) Shade in  $\frac{1}{3}$  of the following strip.



OR

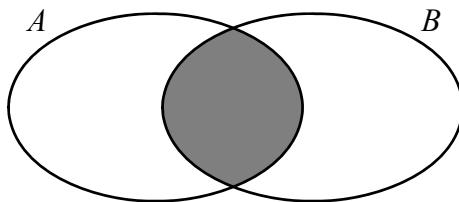


(ii) Fill in the two blanks below, to show two fractions that have the same value as  $\frac{1}{3}$ .

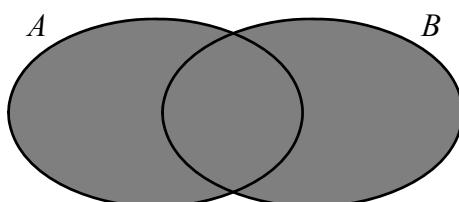
$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

**Question 2****20 Marks**

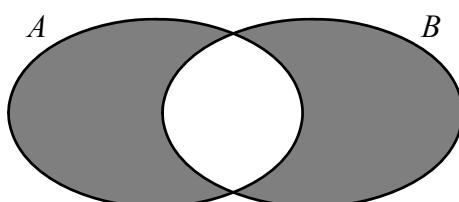
- (a) On the Venn diagram below, **shade in** the region that represents  $A \cap B$ .



- (b) On the Venn diagram below, **shade in** the region that represents  $A \cup B$ .



- (c) On the Venn diagram below, **shade in** the region that represents  $(A \cup B) \setminus (A \cap B)$ .



- (d) Put a tick ( $\checkmark$ ) in the correct box to show which of the following represents the elements that are **in A but not in B**.

$B \setminus A$

$A + B$

$A \setminus B$

**Question 3****25 Marks**

Daniel wants to buy a bike. The usual price of the bike is €320.  
The bike is on “special offer” in three different shops.

- (a) **Shop A** offers **10% off** the usual price of the bike.

Fill in the table to show the “special offer” price of the bike in this shop.

$$10\% = €320 \times 0.10 = €32$$

$$\Rightarrow 90\% = 320 - 32 = €288$$

“Special offer” price: €288.

- (b) **Shop B** offers  **$\frac{1}{4}$  off** the usual price of the bike.

Fill in the table to show the “special offer” price of the bike in this shop.

$$\frac{1}{4} = €320 \times \frac{1}{4} = €80$$

$$\Rightarrow \text{Rest} = 320 - 80 = €240$$

“Special offer” price: €240.

- (c) In **Shop C**, Daniel can pay **€60 now, plus €20 at the end of each month for 12 months**.

Fill in the table to show the “special offer” price of the bike in this shop.

$$60 + (20 \times 12) = 60 + 240$$

$$= €300$$

“Special offer” price: €300.

- (d) Do you think Daniel should buy the bike in shop **A**, **B**, or **C**?

Give a reason for your answer.

Answer: Shop B

Reason: Cheapest at €240

**OR**

Answer: Shop C

Reason: Doesn't have to pay all the money now.

**Question 4****25 Marks**

For her birthday, Rachael went to Belfast with her family.

They left Dublin at 2:50 p.m. and arrived in Belfast 2 hours and 20 minutes later.

- (a) At what time did they arrive in Belfast?

5:10 p.m.

The hotel room cost £140 sterling. The exchange rate was £1 sterling = €1·28.

- (b) Find the cost of the hotel room, in euro (€).

$$140 \times 1\cdot28 = €179\cdot20$$

The family went to a concert in Belfast.

An adult's ticket cost €80. A child's ticket cost €60.

- (c) Write the cost of a child's ticket as a percentage of the cost of an adult's ticket.

$$\frac{60}{80} \times 100 = 75\%.$$

There were 4000 people at the concert.

The ratio of children to adults at the concert was 3:5.

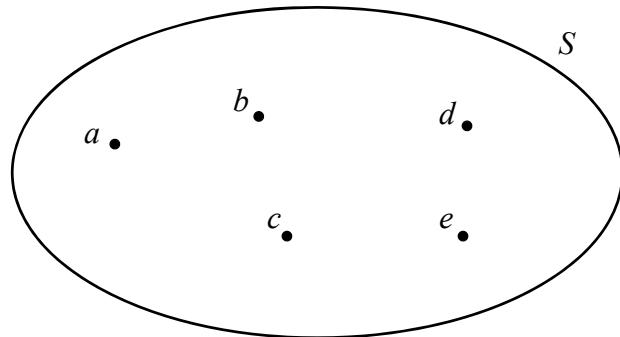
- (d) Find the number of children at the concert.

$$3 + 5 = 8.$$

$$\frac{3}{8} \times 4000 = 1,500.$$

**Question 5****10 Marks**

The set  $S$  is shown in the Venn diagram below. It has 5 elements.



Some students are asked to write down **subsets** of  $S$  that have **3 elements** each.  
Eoin writes down the subset  $\{a, c, d\}$ .

- (a) Write down **two more subsets** of  $S$  that have 3 elements each.

$$\text{Subset 1} = \{a, b, e\}$$

$$\text{Subset 2} = \{b, c, e\}$$

Cliodhna writes down  $\{a, b, w\}$ .

- (b) Explain why this is **not** a subset of  $S$ .

$w$  is not an element of  $S$ .

**Question 6****20 Marks**

A juice bar makes smoothies in two sizes, small and large.  
Their menu is shown below.

Smoothie	Small	Large
Strawberry Slurp	€2·00	€4·00
Banana Boost	€1·50	€3·00
Apple Swirl	€1·80	€3·60
Lemon Crush	€2·10	€4·20

Gary buys a **small** Lemon Crush and a **large** Apple Swirl.

- (a) Find the total cost of these two smoothies.

$$2\cdot10 + 3\cdot60 = €5\cdot70.$$

Elaine wants to buy two small smoothies and one large smoothie. She has €7 to spend.

- (b) Complete the sentence to show one combination of smoothies that Elaine could buy.  
Find the total cost of these three smoothies.

Small Strawberry Slurp, small **Banana Boost**, and large **Banana Boost**.

$$\text{Cost} = 2\cdot00 + 1\cdot50 + 3\cdot00 = €6\cdot50.$$

**OR**

Small Strawberry Slurp, small **Apple Swirl**, and large **Banana Boost**.

$$\text{Cost} = 2\cdot00 + 1\cdot80 + 3\cdot00 = €6\cdot80.$$

**OR**

Small Strawberry Slurp, small **Strawberry Slurp**, and large **Banana Boost**.

$$\text{Cost} = 2\cdot00 + 2\cdot00 + 3\cdot00 = €7\cdot00.$$

The juice bar makes another smoothie, an Orange Twist.

A **small** Orange Twist costs €1·60.

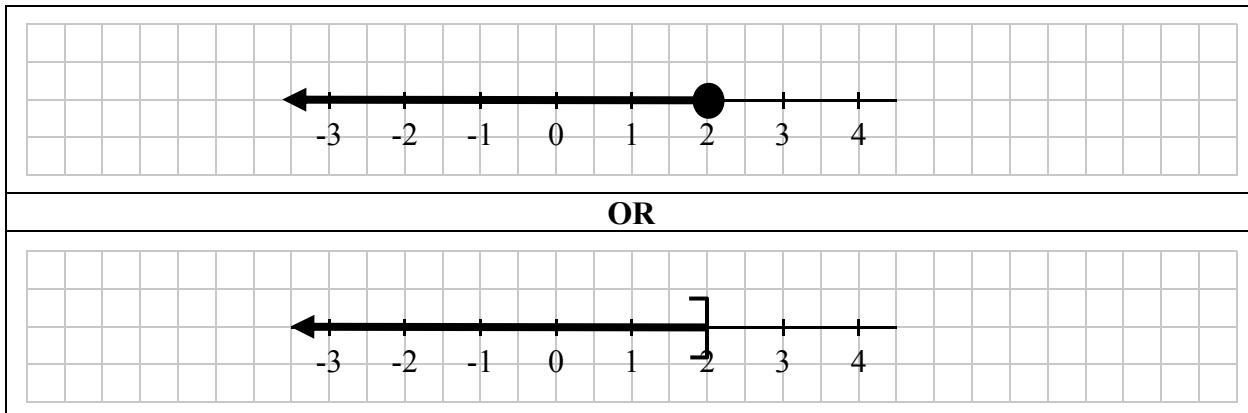
- (c) Use the prices in the menu above to work out how much a **large** Orange Twist costs.  
There is a relationship between the prices of the small and large smoothies in the menu.

$$1\cdot60 \times 2 = €3\cdot20.$$

**Question 7****20 Marks**

- a) Graph the following inequality on the number line below.

$$x \leq 2, \quad x \in \mathbb{R}$$



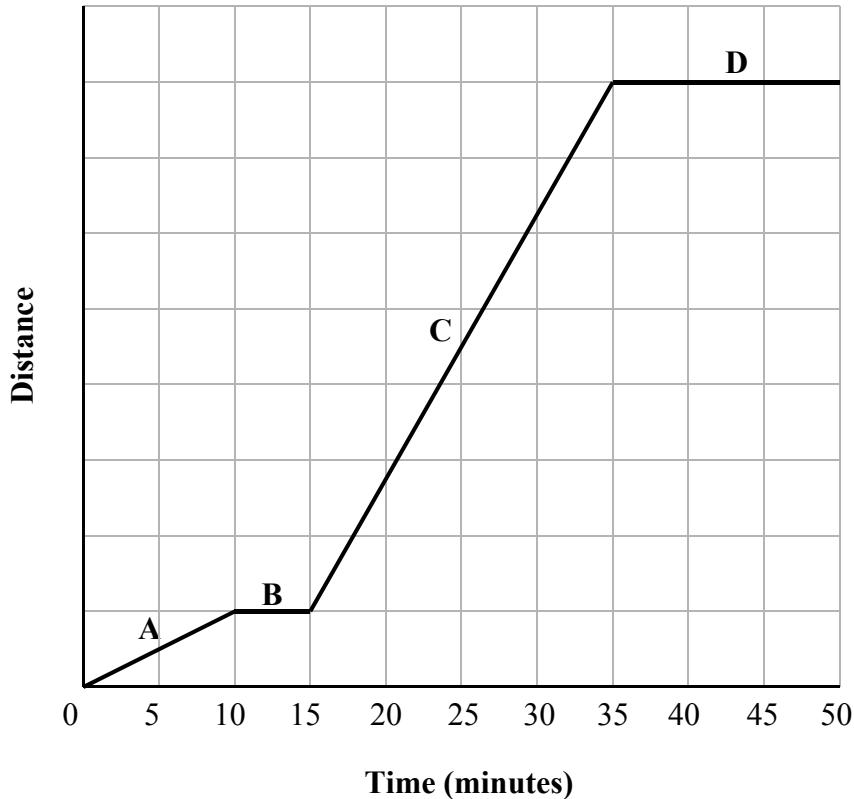
- (b) Put a tick (✓) in the correct box in the table to show which inequality is graphed on the number line below.



Inequality	Put a tick (✓) in <b>one</b> box only
$x \leq 1, \quad x \in \mathbb{N}$	
$x \geq 1, \quad x \in \mathbb{N}$	✓
$x > 1, \quad x \in \mathbb{N}$	
$x < 1, \quad x \in \mathbb{N}$	

**Question 8****20 Marks**

- (a) Gráinne is taking part in a training session.  
The graph shows the distance she travelled during the session.  
The four parts of the graph are labelled **A**, **B**, **C**, and **D**.



- (a) Write the letters **A**, **B**, **C**, and **D** into the table to match each description with the correct part of the graph.

Description	Part of the Graph
Gráinne runs for 20 minutes	<b>C</b>
Gráinne stops for 15 minutes	<b>D</b>
Gráinne walks for 10 minutes	<b>A</b>
Gráinne stops for 5 minutes	<b>B</b>

- (b) Gráinne runs 4 km in 20 minutes at a steady pace.  
Find her speed in km per hour.

$$\text{Speed} = \text{Distance} \div \text{Time} = 12 \text{ km/h.}$$

**OR**

$$4\text{km in } 20 \text{ minutes}$$

$$\Rightarrow 12 \text{ km in } 60 \text{ minutes} = 12 \text{ km/h.}$$

**Question 9****40 Marks**

Factorise fully each of the following.

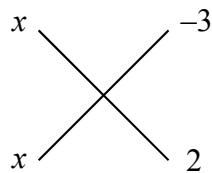
(a)  $7x - 21y$

$$7(x - 3y)$$

(b)  $x^2 - 25$

$$x^2 - 5^2 = (x + 5)(x - 5)$$

(c)  $x^2 - x - 6$



Answer:  $(x - 3)(x + 2)$

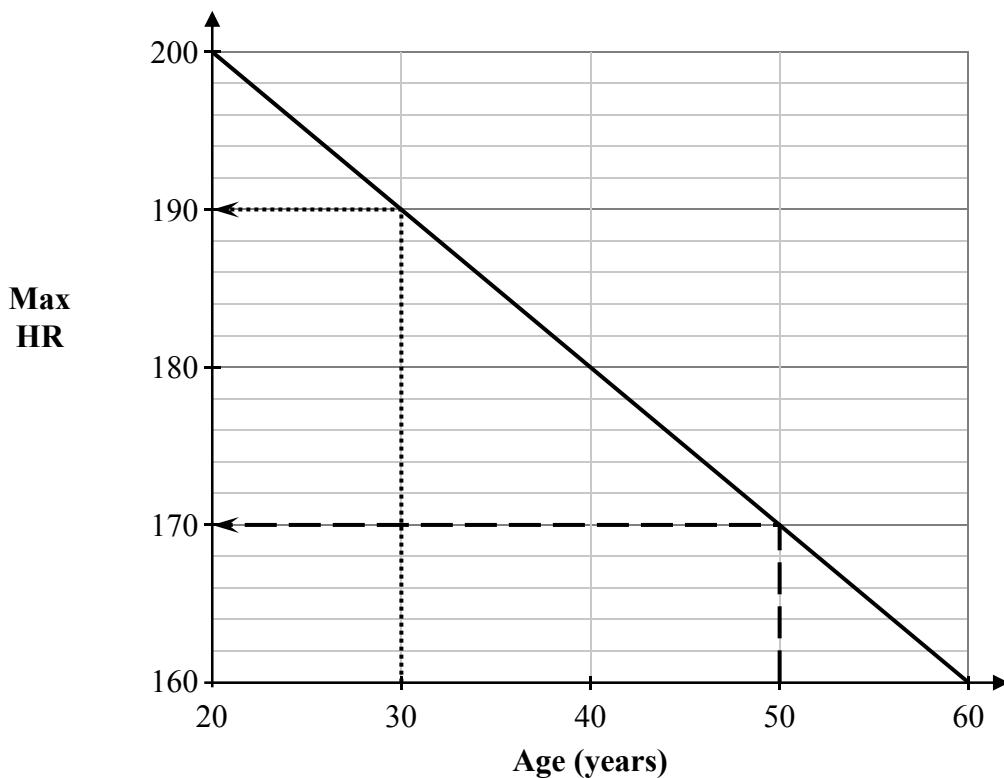
**OR**

$$\begin{aligned} x^2 - x - 6 &= x^2 - 3x + 2x - 6 \\ &= x(x - 3) + 2(x - 3) \\ &= (x + 2)(x - 3) \end{aligned}$$

**Question 10****35 Marks**

A gym has three different formulas to estimate your maximum heart rate (Max HR), given your age in years. Different formulas can give different estimates.

The **first formula** is shown in the graph below.



- (a) Use the graph above to find the Max HR for someone aged 30 years and someone aged 50 years. Show your work on the graph.

Max HR for 30 years = 190

Max HR for 50 years = 170

[See working out on graph.]

- (b) Part of the formula that gives this graph is shown below.  
Fill in the missing number in the formula.

Max HR = 220 minus your Age

The **second formula** for finding Max HR is:

$$\text{Max HR} = 210 \text{ minus Half your Age.}$$

- (c) Use this formula to find the Max HR for someone aged 60 years.

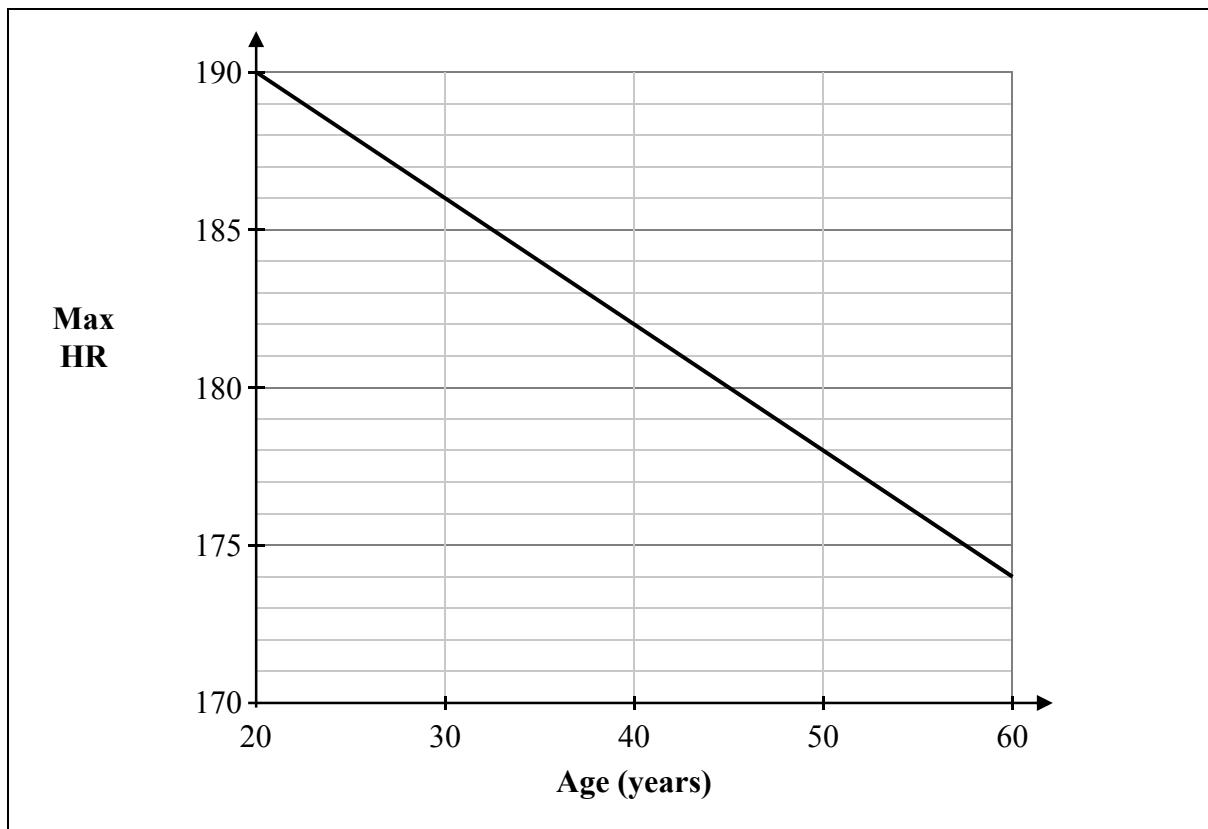
Max HR	$= 210 - 30$ $= 180$
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The **third formula** is shown in the table on the right.  
The pattern in the Max HR column is a **linear** pattern.

- (d) Complete the table.

Age (years)	Max HR
20	190
30	186
40	<b>182</b>
50	<b>178</b>
60	<b>174</b>

- (e) Using the values in the table, draw a graph on the grid below to show the Max HR for all ages from 20 years to 60 years.



**Question 11****40 Marks**

- (a) Solve the equation  $5x - 10 = 3x + 2$ .

$$5x - 10 = 3x + 2$$

$$5x - 3x = 2 + 10$$

$$2x = 12$$

$$x = 6.$$

- (b) John says that  $x = 4$  is a solution of  $x^2 - 2x - 8 = 0$ . Show that John is correct.

$$\begin{aligned} \text{Sub in: } (4)^2 - 2(4) - 8 &= 0 \\ \Rightarrow 16 - 16 &= 0 \end{aligned}$$

**OR**

$$\begin{aligned} x^2 - 2x - 8 &= 0 \\ \Rightarrow (x + 2)(x - 4) &= 0 \\ \Rightarrow x = 4 &[ \text{ or } x = -2 ] \end{aligned}$$

**OR**

**Shows** long division:  $x - 4$  divides in evenly to  $x^2 - 2x - 8$ .

- (c) Solve the simultaneous equations:

$$\begin{aligned} x + y &= 11 \\ x - y &= -5. \end{aligned}$$

$$\begin{array}{r} x + y = 11 \\ x - y = -5 \\ \hline 2x = 6 \\ \Rightarrow x = 3 \\ \Rightarrow y = 8 \end{array}$$

**OR**

Second equation:  $x = y - 5$

Sub into first equation:  $y - 5 + y = 11$

$$\begin{aligned} \Rightarrow 2y &= 16 \\ \Rightarrow y &= 8 \\ \Rightarrow x &= 3 \end{aligned}$$

**Question 12****20 Marks**

Martin creates a pattern of numbers using the instructions in the table below.  
The first number is filled in.

- (a) Complete the table.

Instruction	First Number	Second Number	Third Number
Starting Number	5	6	7
Multiply by 3	$5 \times 3$	<b><math>6 \times 3</math></b>	<b><math>7 \times 3</math></b>
Subtract 5 from your answer	$15 - 5$	<b><math>18 - 5</math></b>	<b><math>21 - 5</math></b>
Outcome	10	<b>13</b>	<b>16</b>

- (b) Martin picks a starting number and, using the instructions, gets an outcome of 1.  
Find the **starting number** he picked.

Trial and Improvement:  $(2 \times 3) - 5 = 1$

Answer = 2.

**OR**

Reverse operations:  $1 + 5 = 6$

$6 \div 3 = 2$

Answer = 2.

**OR**

Outcomes form linear pattern:  $16 \rightarrow 13 \rightarrow 10 \rightarrow 7 \rightarrow 4 \rightarrow 1$

Corresponding starting numbers:  $7 \rightarrow 6 \rightarrow 5 \rightarrow 4 \rightarrow 3 \rightarrow 2$

Answer = 2.

- (c) When the starting number is  $k$ , what is the **outcome**? Give your answer in terms of  $k$ .

$3k - 5$ .

# **Marking Scheme – Paper 1**

## **Structure of the marking scheme**

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect), scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5-mark scale	0, 5	0, 2, 5	0, 2, 3, 5	
10-mark scale		0, 5, 10	0, 4, 7, 10	0, 3, 5, 8, 10
15-mark scale		0, 5, 15	0, 5, 10, 15	0, 4, 8, 12, 15
20-mark scale			0, 5, 12, 20	0, 5, 10, 15, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

## **Marking scales – level descriptors**

### **A-scales (two categories)**

- incorrect response (no credit)
- correct response (full credit)

### **B-scales (three categories)**

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

### **C-scales (four categories)**

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

### **D-scales (five categories)**

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may be awarded. Thus, for example, in Scale 10C, 9 marks may be awarded.

No marks may be awarded other than those on the appropriate scale, and *Full Credit –1*.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

## **Summary of mark allocations and scales to be applied**

### **Question 1 (25)**

- (a)(i)–(iii) 10D  
(b) 5C  
(c)(i)&(ii) 10C

### **Question 5 (10)**

- (a) 5C  
(b) 5A

### **Question 9 (40)**

- (a) 10B  
(b) 15B  
(c) 15C

### **Question 2 (20)**

- (a)&(b) 10B  
(c) 5A  
(d) 5B

### **Question 6 (20)**

- (a) 5B  
(b) 10C  
(c) 5B

### **Question 10 (35)**

- (a)&(b) 15C  
(c) 5B  
(d) 5B

### **Question 3 (25)**

- (a)–(c) 20D  
(d) 5B

### **Question 7 (20)**

- (a)&(b) 20C

### **Question 11 (40)**

- (a) 10B  
(b) 10B  
(c) 20C

### **Question 4 (25)**

- (a) 5B  
(b) 5B  
(c)&(d) 15C

### **Question 8 (20)**

- (a) 10D  
(b) 10B

### **Question 12 (20)**

- (a) 10C  
(b)&(c) 10C

## **Detailed marking notes**

### **Question 1 (25 Marks)**

#### **(a)(i)–(iii) Scale 10D (0,3 ,5 ,8, 10 )**

*Low Partial Credit*

- One correct operation

*Mid Partial Credit*

- One correct answer

*High Partial Credit*

- Two correct answers

#### **(b) Scale 5C (0, 2, 3, 5 )**

Tolerance: < 1 box

*Low Partial Credit*

- One correct answer

*High Partial Credit*

- Two correct answers

*Full Credit –I*

- No labels

*Full Credit*

- Labels filled in accurately, but points not indicated

#### **(c)(i)&(ii) Scale 10C (0, 4, 7, 10 )**

*Low Partial Credit*

- Shading correct, **or** one fraction correct

*High Partial Credit*

- Both fractions correct, **or** shading and one fraction correct

## **Question 2 (20 Marks)**

### **(a)&(b) Scale 10B (0, 5, 10)**

*Partial Credit*

- (a) or (b) correct, **or** mixes up  $\cap$  and  $\cup$

### **(c) Scale 5A (0, 5)**

*Full Credit*

- May be awarded if answer is consistent with (a) and (b), as long as there is some shading

### **(d) Scale 5B (0, 2, 5)**

*Partial Credit*

- $B \setminus A$  ticked, **or** 2 boxes ticked to include correct answer

## **Question 3 (25 Marks)**

### **(a)–(c) Scale 20D (0, 5, 10, 15, 20)**

*Low Partial Credit*

- One relevant calculation

*Mid Partial Credit*

- One correct answer

*High Partial Credit*

- Two correct answers

*Full Credit*

- Accept correct answers without € sign

### **(d) Scale 5B (0, 2, 5)**

*Partial Credit*

- Answer **or** reason correct (Answer = Shop C, or cheaper of A and B)

*Full Credit*

- Reason must correspond to answer given for *Full Credit*

#### **Question 4 (25 Marks)**

- (a) **Scale 5B (0, 2, 5)**

*Partial Credit*

- One correct step

- (b) **Scale 5B (0, 2, 5)**

*Partial Credit*

- $140 \times 1.28$  or  $140 \div 1.28$

- (c)&(d) **Scale 15C (0, 5, 10, 15)**

*Low Partial Credit*

- In (c): use of 60 or 80 or 100, or answer of 25%
- In (d): finds 8

*High Partial Credit*

- (d) correct

#### **Question 5 (10 Marks)**

- (a) **Scale 5C (0, 2, 3, 5)**

*Low Partial Credit*

- Any element in  $S$  listed, **or** permutations of  $\{a, c, d\}$  used in both boxes

*High Partial Credit*

- One correct new subset

*Full Credit –I*

- Misreading: uses the set  $\{a, c, d\}$  (or a permutation of it), and a different correct subset of  $S$ .

- (b) **Scale 5A (0, 5)**

*Full Credit*

- Accept “ $w$  not in it”, “because it’s not in the Venn diagram”, and similar.

### **Question 6 (20 Marks)**

**(a) Scale 5B (0, 2, 5)**

*Partial Credit*

- Use of a number from the table

*Full Credit –I*

- Correct answer but no unit

**(b) Scale 10C (0, 4, 7, 10)**

*Low Partial Credit*

- Fills in sentence correctly, **or** use of a number from the table

*High Partial Credit*

- Use of 2 small and 1 large which cost more than €7
- Works out price correctly but doesn't fill in sentence

*Full Credit –I*

- Correct answer but no unit (if first time unit is missing in this question)

**(c) Scale 5B (0, 2, 5)**

*Partial Credit*

- Investigates relationship between small and large prices, **or** use of 1·60

*Full Credit –I*

- Correct answer but no unit (if first time unit is missing in this question)

### **Question 7 (20 Marks)**

**(a)&(b) Scale 20C (0, 5, 12, 20)**

*Low Partial Credit*

- Relevant work in **either** (a) **or** (b). Accept  $x > 1$  ticked as relevant work in (b).

*High Partial Credit*

- (a) correct

*Full Credit*

- Arrow not needed in (a)

### **Question 8 (20 Marks)**

#### **(a) Scale 10D (0, 3, 5, 8, 10)**

If there are 2 or more uncancelled letters in any box, that box is marked incorrect.

*Low Partial Credit*

- One correct

*Mid Partial Credit*

- Two correct

*High Partial Credit*

- Three correct

#### **(b) Scale 10B (0, 5, 10)**

*Partial Credit*

- Use of distance or time

### **Question 9 (40 Marks)**

#### **(a) Scale 10B (0, 5, 10)**

*Partial Credit*

- One factor of solution correct

#### **(b) Scale 15B (0, 5, 15)**

*Partial Credit*

- A correct factorising of  $x^2$  or 25, or one factor of solution correct

#### **(c) Scale 15C (0, 5, 10, 15)**

*Low Partial Credit*

- Any correct factorising of 6 or  $x^2$
- Attempt at expansion, i.e.  $x^2 - 3x + 2x - 6$
- Any correct substitution into  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

*High Partial Credit*

- One factor from solution correct
- Answer multiplies out to give 2 correct terms
- 2 roots found from quadratic formula
- Cross filled in correctly

**Question 10 (35 Marks)**

**(a)&(b) Scale 15C (0, 5, 10, 15)**

Tolerance:  $\pm 1$  unit

*Low Partial Credit*

- One value correct or relevant work on graph

*High Partial Credit*

- Two values correct

*Full Credit –1*

- Misreading: Work not shown on the graph

**(c) Scale 5B (0, 2, 5)**

*Partial Credit*

- Some element of formula used correctly

**(d) Scale 5B (0, 2, 5)**

*Partial Credit*

- Work relevant to continuing linear pattern

**(e) Scale 10C (0, 4, 7, 10)**

Tolerance:  $< 1$  unit on vertical axis

*Low Partial Credit*

- Two points correctly plotted

*High Partial Credit*

- Five points correctly plotted with no / incorrect line
- Bar chart drawn correctly, consistent with values in (d)

**Question 11 (40 Marks)**

- (a) **Scale 10B (0, 5, 10)**

*Partial Credit*

- One relevant operation

- (b) **Scale 10B (0, 5, 10)**

*Partial Credit*

- One correct substitution
- One correct step in solving equation given (including writing down quadratic formula)
- Sets up long division

- (c) **Scale 20C (0, 5, 12, 20)**

*Low Partial Credit*

- One correct operation
- Substitution of solution into one equation
- Substitution of incorrect solution into both equations

*High Partial Credit*

- One correct solution for  $x$  or  $y$ , by algebra

*Full Credit*

- Solution by inspection, or by trial and improvement, needs to be verified for both equations

**Question 12 (20 Marks)**

(a) **Scale 10C (0, 4, 7, 10 )**

*Low Partial Credit*

- One correct entry

*High Partial Credit*

- Four correct entries

*Full Credit*

- Can write 18 instead of  $6 \times 3$ , etc.

(b)&(c) **Scale 10C (0, 4, 7, 10)**

*Low Partial Credit*

- Relevant work in **either (b) or (c)**:  
In (b): e.g. uses  $k$ , or some trial and improvement  
In (c): some relevant use of  $k$

*High Partial Credit*

- (b) or (c) correct, **or** relevant work in (b) **and** (c), as described above





Coimisiún na Scrúduithe Stáit  
State Examinations Commission

Junior Certificate Examination 2015

# Mathematics

Paper 2

Ordinary Level

## Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

## **Instructions**

There are 12 questions on this examination paper. Answer **all** questions.

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

**You will lose marks if all necessary work is not clearly shown.**

**You may lose marks if the appropriate units of measurement are not included, where relevant.**

**You may lose marks if your answers are not given in simplest form, where relevant.**

Write the make and model of your calculator(s) here:

**Question 1****20 Marks**

The songs on Gavin's phone are shown in the table below.

Singer	Number of songs
Usher	
Pharrell	15
Ed Sheeran	4
Hozier	3

Gavin has **30 songs** on his phone, in total.

- (a) Find how many songs by Usher are on Gavin's phone.

$$30 - (15 + 4 + 3) = 30 - 22 \\ = 8.$$

Gavin plays a song at random on his phone.

- (b) Find the **probability** that this song is by Hozier.

$$\frac{3}{30} \text{ or } \frac{1}{10}.$$

- (c) Find the **probability** that this song is by Ed Sheeran **or** Pharrell.

$$\frac{15+4}{30} = \frac{19}{30}.$$

Gavin plays a song by Ed Sheeran, and then plays a song by Hozier.

- (d) In **how many different ways** can he do this?

Remember that he has 4 songs by Ed Sheeran and 3 songs by Hozier.

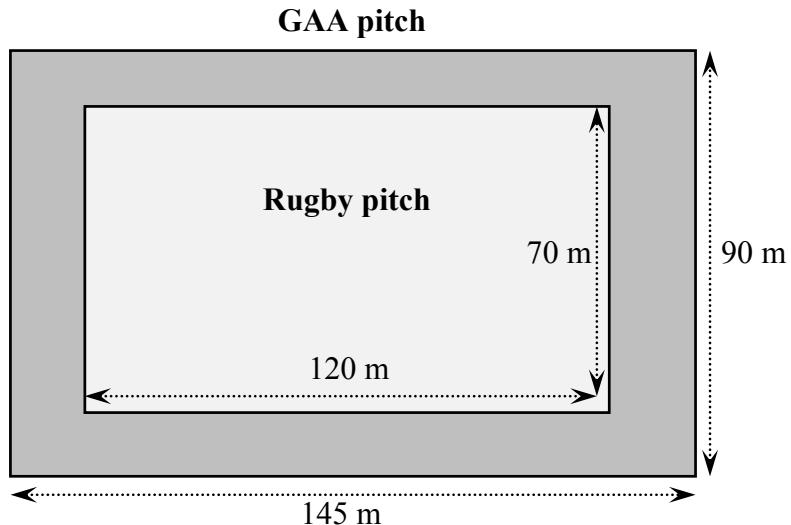
$$3 \times 4 = 12.$$

**Question 2****20 Marks**

When the Irish rugby team played in Croke Park, a rugby pitch was made inside the GAA pitch.

The GAA pitch was 145 m long and 90 m wide.

The rugby pitch was 120 m long and 70 m wide.



- (a) Find the **area** of each pitch.

Area of GAA pitch:

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= 145 \times 90 \\ &= 13,050 \text{ m}^2. \end{aligned}$$

Area of rugby pitch:

$$\begin{aligned} \text{Area} &= \text{length} \times \text{width} \\ &= 120 \times 70 \\ &= 8,400 \text{ m}^2. \end{aligned}$$

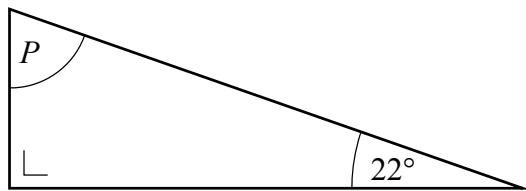
- (b) What area of the GAA pitch was **not** used for rugby?

$$13,050 - 8,400 = 4,650 \text{ m}^2.$$

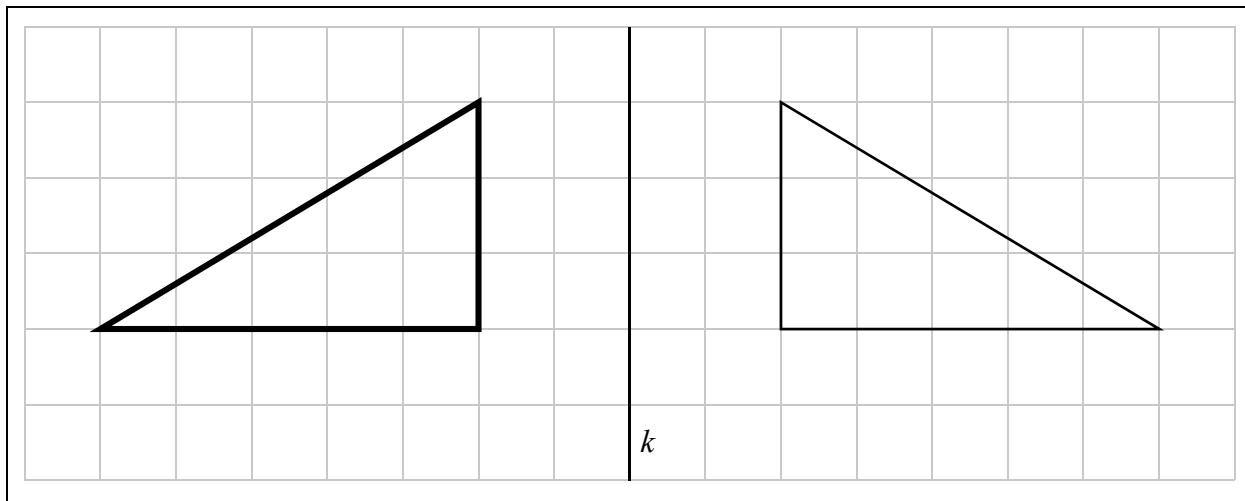
**Question 3****30 Marks**

- (a) Calculate the size of the angle marked  $P$  in the right-angled triangle below.

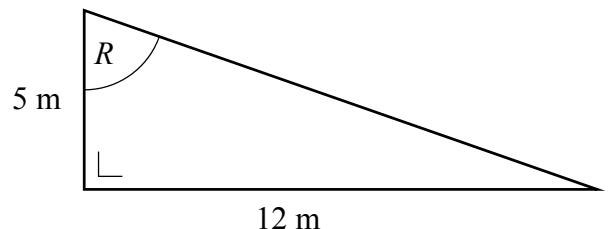
$$180 - 90 - 22 = 68^\circ.$$



- (b) Draw the image of the triangle below under **axial symmetry** in the line  $k$ .



- (c) (i) Write down the length of the side **opposite** the angle  $R$  in the triangle shown.



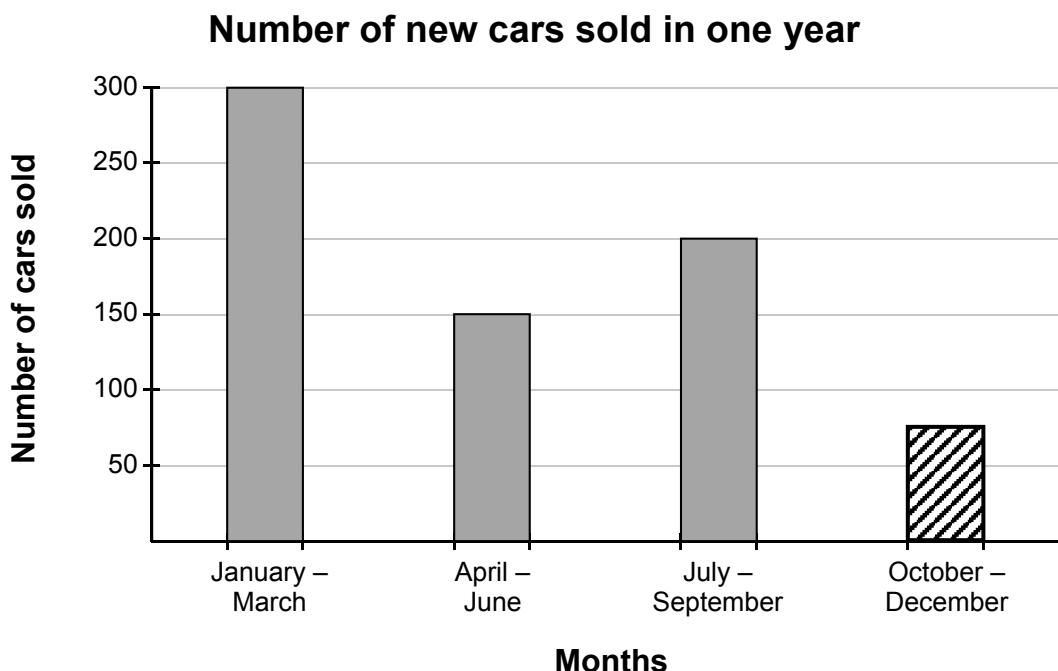
$$\text{Opposite} = 12 \text{ m.}$$

- (ii) Use the Theorem of Pythagoras to find the length of the **hypotenuse** of this triangle.

$$\begin{aligned}\sqrt{5^2 + 12^2} &= \sqrt{25 + 144} \\ &= \sqrt{169} \\ &= 13 \text{ m.}\end{aligned}$$

**Question 4****25 Marks**

The diagram shows the number of new cars sold in a garage in one year.



- (a) How many new cars were sold in the months **April – June**?

150.

In the months **October – December**, there were exactly **half** as many new cars sold as in April – June.

- (b) How many new cars were sold in **October – December**?

$$150 \div 2 = 75.$$

- (c) Draw the bar for **October – December** on the diagram above.

[See diagram above.]

- (d) When were the **most** new cars sold? Put a tick (✓) in the correct box.

January – March	April – June	July – September	October – December
<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

- (e) Calculate the **total** number of new cars sold in the year.

$$300 + 150 + 200 + 75 = 725.$$

- (f) Calculate the **average** (mean) number of new cars sold **per month** in the year.  
Give your answer correct to one decimal place.

$$\begin{aligned} 725 \div 12 &= 60.416666\ldots \\ &= 60.4 \text{ (1 decimal place).} \end{aligned}$$

### Question 5

**15 Marks**

- (a) One of the following is a description of the **mode** of a list.  
Put a tick (✓) in the correct box to show which one.

Description	Put a tick (✓) in <b>one</b> box
The <b>middle</b> value in an ordered list	
The <b>biggest</b> value in a list	
The <b>most common</b> value in a list	✓

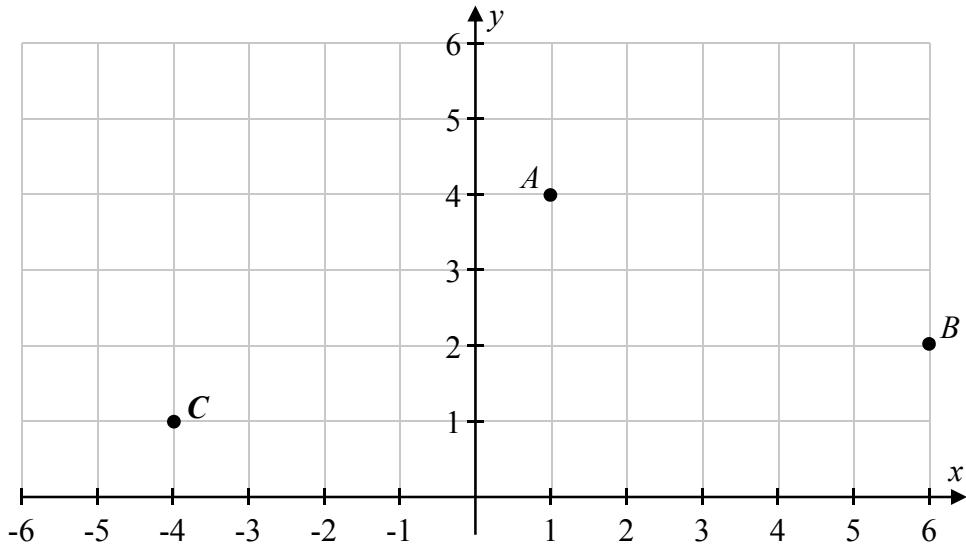
- (b) Write out a list that has a mode, and write down the mode of your list.

List: Bob, Bob, Hugo.

Mode: Bob.

**Question 6****40 Marks**

The points  $A$  and  $B$  are shown on the co-ordinate grid below.



- (a) Write down the co-ordinates of the point  $A$ .

$$A = (1, 4).$$

$B$  is the point  $(6, 2)$ .

- (b) Find the **length** of  $[AB]$ . Give your answer in the form  $\sqrt{x}$ , where  $x \in \mathbb{N}$ .

$$\begin{aligned}\sqrt{(6-1)^2 + (2-4)^2} &= \sqrt{(5)^2 + (-2)^2} \\ &= \sqrt{25+4} \\ &= \sqrt{29}.\end{aligned}$$

$C$  is the point  $(-4, 1)$ .

- (c) Plot the point  $C$  on the co-ordinate grid above.  
Label the point  $C$  clearly.

See diagram above.

- (d) Find the **slope** of the line  $CA$ .

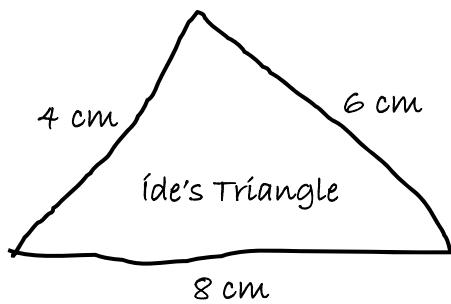
$$\frac{\text{rise}}{\text{run}} = \frac{3}{5}$$

OR

$$m = \frac{4-1}{1-(-4)} = \frac{3}{5}$$

**Question 7****25 Marks**

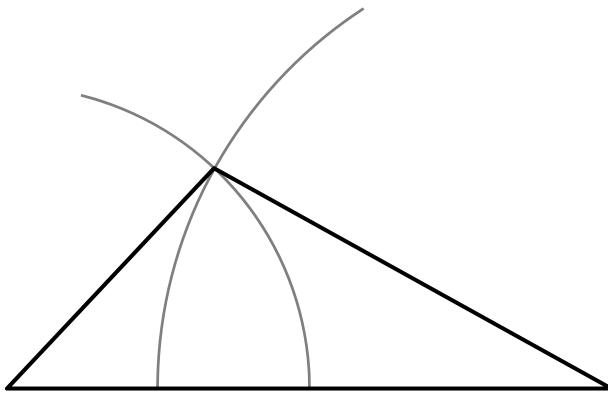
Íde draws the sketch of the triangle shown. The lengths of the sides are 4 cm, 6 cm, and 8 cm.



- (a) What type of triangle has Íde sketched? Put a tick ( $\checkmark$ ) in the correct box.

Type of Triangle	Put a tick ( $\checkmark$ ) in one box
Isosceles	
Scalene	$\checkmark$
Equilateral	

- (b) Construct Íde's triangle in the box below. Show your construction lines clearly.



- (c) Measure the biggest angle in your triangle from part (b).

Write the size of this angle into the box below, correct to the nearest degree.

Size of biggest angle =  $104^\circ$  or  $105^\circ$

**Question 8****30 Marks**

The marks that 9 students got on a test are:

23	16	13	30	26	15	18	23	20
----	----	----	----	----	----	----	----	----

- (a) Write out all 9 marks **in order**, from the smallest to the biggest.

Answer = 

13	15	16	18	20	23	23	26	30
----	----	----	----	----	----	----	----	----

- (b) Write down the **median** mark.

Median mark = 20

- (c) Find the **range** of the marks.

$$\begin{aligned}\text{Maximum} - \text{Minimum} &= 30 - 13 \\ &= 17.\end{aligned}$$

The teacher **adds 2 marks** onto each student's mark.

- (d) Find the **new range** of the marks.

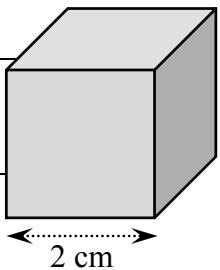
$$\begin{aligned}\text{New Max} - \text{New Min} &= 32 - 15 \\ &= 17.\end{aligned}$$

**Question 9****25 Marks**

A cube has sides of length 2 cm.

- (a) Find the **volume** of the cube.

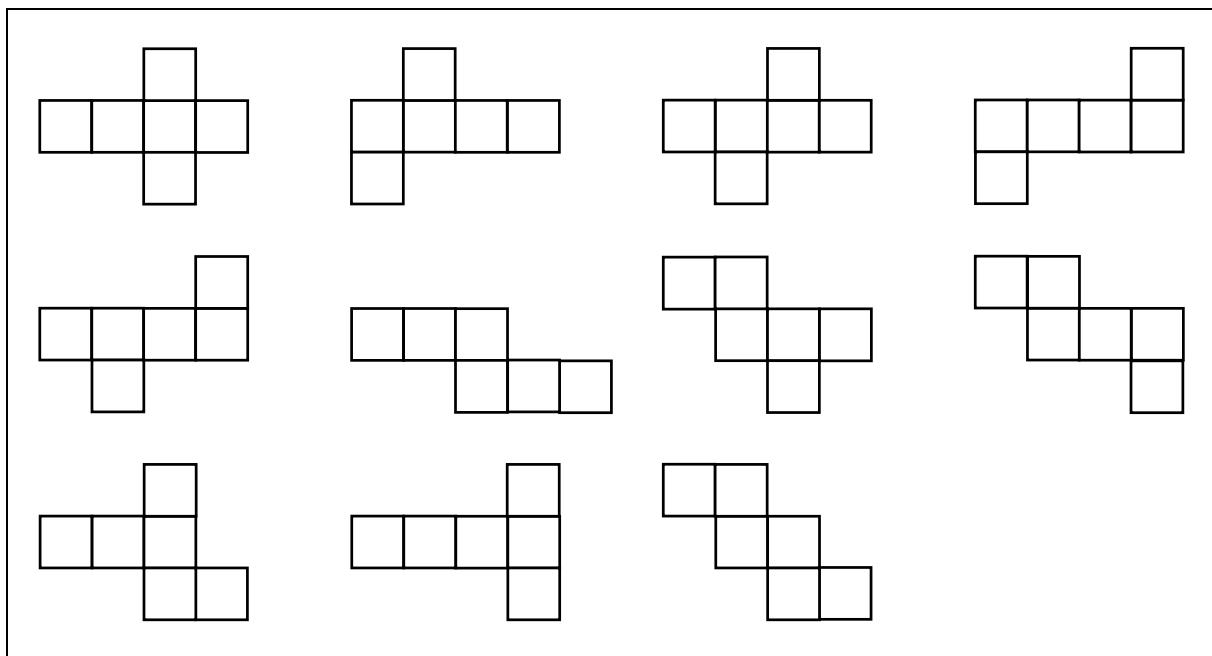
$$2 \times 2 \times 2 = 8 \text{ cm}^3.$$



- (b) How many **faces** does a cube have?

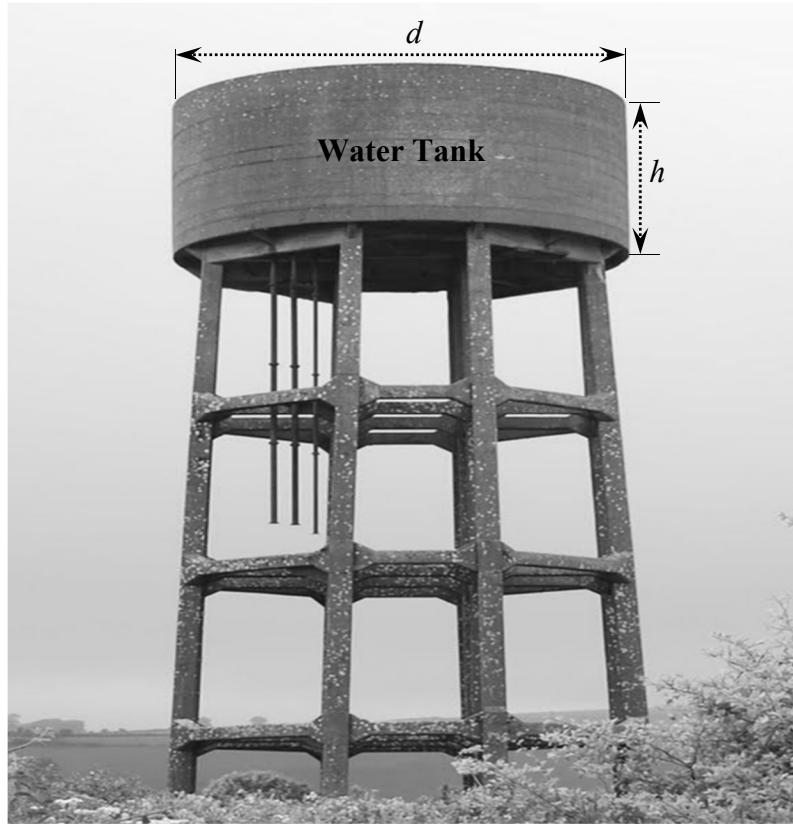
6.

- (c) Draw a **net** of the cube, as accurately as you can.



**Question 10****30 Marks**

The photograph shows a water tank in the shape of a cylinder. The height ( $h$ ) and diameter ( $d$ ) of the tank are marked.



Source: www.watertowersofireland.com. Altered.

- (a) Using your **ruler**, find the height and the diameter of the tank in the photograph. Give each answer correct to the nearest centimetre.

$$h = 2 \text{ cm.}$$

$$d = 6 \text{ cm.}$$

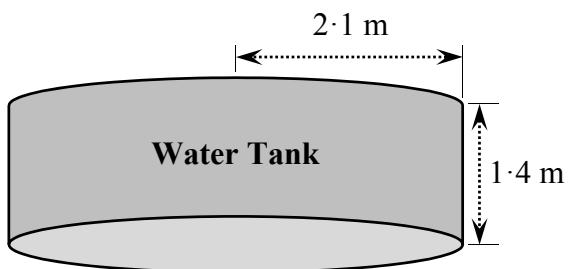
Jenny thinks that the **actual height** of the water tank is 1 m.

- (b) Use Jenny's value to find the **actual diameter** and the **actual radius** of the tank. Give each answer in metres.

$$\begin{aligned} \text{Actual diameter} &= 6 \div 2 \\ &= 3 \text{ m.} \end{aligned}$$

$$\begin{aligned} \text{Actual radius} &= 3 \div 2 \\ &= 1.5 \text{ m.} \end{aligned}$$

Colm finds other values for the actual height and the actual radius of the tank. They are shown in the diagram below.



- (c) Use Colm's values to find the **volume** of the tank.  
Give your answer in  $\text{m}^3$ , correct to one decimal place.

$$\begin{aligned}\text{Volume} &= \pi r^2 h \\ &= \pi \times 2\cdot1 \times 2\cdot1 \times 1\cdot4 \\ &= 19\cdot38636\dots \\ &= 19\cdot4 \text{ m}^3 \text{ (1 decimal place).}\end{aligned}$$

### Question 11

**20 Marks**

- (a) Use your calculator to find the value of each of the following.  
Give each answer correct to two decimal places.

(i)  $\sin 20^\circ =$   and  $\cos 70^\circ =$

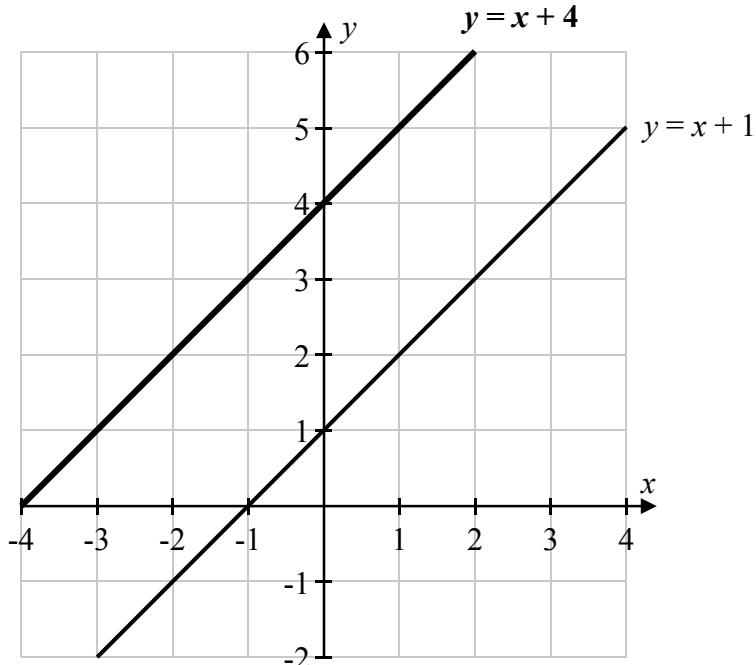
(ii)  $\sin 50^\circ =$   and  $\cos 40^\circ =$

- (b) Hence, or otherwise, fill in the correct angle below.

$$\sin 10^\circ = \cos [80]^\circ$$

**Question 12****20 Marks**

The graph of the line  $y = x + 1$  is shown on the co-ordinate grid below.



- (a) Write down the co-ordinates of the point where this line crosses the **y-axis**.

( 0 , 1 )

- (b) Write down the co-ordinates of the point where the line  $y = x + 4$  crosses the **y-axis**.

( 0 , 4 )

- (c) Hence, or otherwise, draw the graph of the line  $y = x + 4$  on the co-ordinate grid above.

[See diagram above.]

# **Marking Scheme – Paper 2**

## **Structure of the marking scheme**

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect), scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D
No of categories	2	3	4	5
5-mark scale	0, 5	0, 3, 5	0, 2, 3, 5	
10-mark scale		0, 5, 10	0, 3, 7, 10	
15-mark scale			0, 4, 8, 15	0, 3, 6, 9, 15
20-mark scale				0, 4, 8, 12, 20

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

## **Marking scales – level descriptors**

### **A-scales (two categories)**

- incorrect response (no credit)
- correct response (full credit)

### **B-scales (three categories)**

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

### **C-scales (four categories)**

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

### **D-scales (five categories)**

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work, or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may be awarded. Thus, for example, in Scale 10C, 9 marks may be awarded.

No marks may be awarded other than those on the appropriate scale, and *Full Credit –1*.

In general, accept a candidate's work in one part of a question for use in subsequent parts of the question, unless this oversimplifies the work involved.

## **Summary of mark allocations and scales to be applied**

### **Question 1 (20)**

- (a) 5C
- (b)&(c) 5C
- (d) 10B

### **Question 5 (15)**

- (a) 5A
- (b) 10B

### **Question 9 (25)**

- (a)&(b) 15C
- (c) 10C

### **Question 2 (20)**

- (a) 15C
- (b) 5B

### **Question 6 (40)**

- (a) 5B
- (b) 15C
- (c) 5B
- (d) 15C

### **Question 10 (30)**

- (a) 10B
- (b) 5B
- (c) 15D

### **Question 3 (30)**

- (a) 5B
- (b) 5C
- (c)(i) 5A
- (c)(ii) 15C

### **Question 7 (25)**

- (a) 5A
- (b)&(c) 20D

### **Question 11 (20)**

- (a)(i)&(ii) 10B
- (b) 10B

### **Question 4 (25)**

- (a)–(c) 5C
- (d) 5A
- (e)&(f) 15C

### **Question 8 (30)**

- (a) 5B
- (b) 5B
- (c) 10C
- (d) 10C

### **Question 12 (20)**

- (a) 5B
- (b)&(c) 15C

## Detailed marking notes

### Question 1 (20 Marks)

(a) Scale 5C (0, 2, 3 ,5 )

*Low Partial Credit*

- Addition of any two relevant numbers
- 30 minus a relevant number

*High Partial Credit*

- Gives 22 as the answer.

(b)&(c) Scale 5C (0, 2 ,3, 5 )

*Low Partial Credit*

- Denominator of 30
- Numerator of 3 ( part (b) )
- Numerator of 19 or 15 or 4 ( part (c) )

*High Partial Credit*

- Part (a) or part (b) correct.

(d) Scale 10B (0, 5 ,10 )

*Partial Credit*

- Multiplication by 4 or 3.
- Attempt at listing

### Question 2 (20 Marks)

(a) Scale 15C (0, 4 ,8 ,15)

*Low Partial Credit*

- Gets correct perimeter of one or both pitches .
- Adds instead of multiplies .
- States area = length  $\times$  breadth .

*High Partial Credit*

- One correct area .

*Full Credit –I*

- Correct answer but no unit

**(b) Scale 5B (0, 3, 5)**

*Partial Credit*

- $13050 - 8400$
- Difference between two lengths or two widths.

*Full Credit –I*

- Correct answer but no unit (if first time unit is missing in this question)

**Question 3 (30 Marks)**

**(a) Scale 5B (0, 3, 5)**

*Partial Credit*

- $112^\circ$  or valid use of  $22^\circ$  or  $90^\circ$
- States the three angles in a triangle sum to  $180^\circ$

*Full Credit*

- Accept solution without unit (degree symbol)

**(b) Scale 5C (0, 2, 3 , 5 )**

*Low Partial Credit*

- One point correct
- A triangle drawn to the left of the axis.

*High Partial Credit*

- Two or three points correct but the triangle is not drawn
- Correct triangle in wrong location , but with correct orientation .

**(c)(i) Scale 5A ( 0 , 5 )**

**(c)(ii) Scale 15C (0, 4 , 8 , 15 )**

*Low Partial Credit*

- $5^2$  or  $12^2$ .
- States Pythagoras' Theorem .

*High Partial Credit*

- Answer of 169 .
- $25 + 144$  or  $5^2 + 12^2$
- 13 with no work ( may be indicated in diagram )

*Full Credit –I*

- Correct answer but no unit

**Question 4 (25 Marks)****(a)–(c) Scale 5C (0, 2, 3 , 5)***Low Partial Credit*

- One part correct .

*High Partial Credit*

- Two parts correct .

**(d) Scale 5A (0, 5)****(e)&(f) Scale 15C (0, 4 , 8 , 15 )***Low Partial Credit*

- Part ( e ) correct

*High Partial Credit*

- Part ( f ) correct

**Question 5 (15 Marks)****(a) Scale 5A (0, 5)****(b) Scale 10B (0, 5, 10)***Partial Credit*

- Gives a list with a repeated element .

**Question 6 (40 Marks)****(a) Scale 5B (0, 3, 5)***Partial Credit*

- $A = ( 4 , 1 )$  or one co-ordinate correct .

**(b) Scale 15C (0, 4 , 8 , 15 )***Low Partial Credit*

- Correct distance formula .
- Correct substitution into slope or midpoint formula .
- $5^2$  or  $( 6 - 1 )^2$  or  $( -2 )^2$  or  $( 2 - 4 )^2$  .
- Indicates length of two sides of right-angled triangle ( 2 and 5 ).

*High Partial Credit*

- Incorrect labelling and finds distance correctly .
- $25 + 4$ .
- $\sqrt{25 - 4}$  .

**(c) Scale 5B (0, 3, 5)**

*Partial Credit*

- Plots point correctly but does not label .
- Plots ( 1 , - 4 ) correctly and labels it correctly .

**(d) Scale 15C (0, 4, 8 , 15)**

*Low Partial Credit*

- A correct slope formula ( including  $\frac{\text{rise}}{\text{run}}$  ) .
- Correct substitution into distance or midpoint formula .

*High Partial Credit*

- Incorrect labelling of points and finds slope consistent with this .
- Finds slope of AB (i.e. oversimplified because of misreading)
- Correct substitution into slope formula but does not finish , e.g.  $\frac{4-1}{1-(-4)}$  .
- $\frac{4-1}{1-4} = -1$  .

**Question 7 (25 Marks)**

**(a) Scale 5A (0, 5)**

**(b)&(c) Scale 20D (0, 4 , 8 , 12 , 20 )**

*Low Partial Credit*

- Side of 8 cm , 6 cm or 4 cm drawn .
- Angle measured correctly .

*Mid Partial Credit*

- One side correct and one or two correct arcs drawn .
- Triangle correct but without arcs and correct angle not found
- Two sides correct with no arc shown and correct angle found .

*High Partial Credit*

- Correct construction but angle not measured or measured incorrectly .
- Angle measured correctly but arcs not shown .
- Two sides correct with arc shown and correct angle found .

**Question 8 (30 Marks)**

**(a) Scale 5B (0, 3 ,5 )**

*Partial Credit*

- 13 is the first entry or 30 is the last entry

**(b) Scale 5B (0, 3, 5)**

*Partial Credit*

- Gives mode ( 23 )
- Gives mean ( 20.44 )
- Gives some relevant explanation of median .

**(c) Scale 10C (0, 3, 7, 10)**

*Low Partial Credit*

- Singles out 13 or 30 .

*High Partial Credit*

- 30 – 13 or states 13 to 30 .

**(d) Scale 10C (0, 3, 7, 10)**

*Low Partial Credit*

- Mention of 32 or 15 .

*High Partial Credit*

- 32 – 15 or 32 to 15 .

**Question 9 (25 Marks)**

**(a)&(b) Scale 15C (0, 4, 8, 15)**

*Low Partial Credit*

- Part ( b ) correct

*High Partial Credit*

- Part ( a ) correct

*Full Credit –I*

- Both parts correct but no unit in (a)

**(c) Scale 10C (0, 3, 7, 10)**

*Low Partial Credit*

- Net drawn that contains squares only .

*High Partial Credit :*

- One square incorrect : extra , missing or in the wrong place .

**Question 10 (30 Marks)****(a) Scale 10B (0 , 5 , 10)***Partial Credit*

- One correct dimension .
- Values of h and d reversed .
- $h = 20 \text{ cm}$  and  $d = 60 \text{ cm}$  .

**(b) Scale 5B (0, 3, 5)***Partial Credit*

- Multiplication by 50 or recognition that scale factor is 50 .
- Attempt at division by 2
- Multiplication by 100 .

**(c) Scale 15D (0, 3, 6 , 9 , 15 )***Low Partial Credit*

- Correct formula ( $\pi r^2 h$ )
- Substituting r or h into  $2\pi rh$  formula .
- Correct multiplication of any two of  $\pi$  , r or h .

*Mid Partial Credit*

- Substitutes  $r = 2.1$  or  $h = 1.4$  into correct formula

*High Partial Credit*

- $\pi \times 2.1 \times 2.1 \times 1.4$  .
- Takes  $r = 1.4$  and  $h = 2.1$  and finishes .
- Takes  $r = 1.5$  and  $h = 2$  ( from previous parts ) and finishes .

**Question 11 (20 Marks)****(a) Scale 10B (0, 5, 10)***Partial Credit*

- One correct answer .
- Calculator in incorrect mode (Rad or Grad)

**(b) Scale 10B (0, 5, 10 )***Partial Credit*

- $\sin 10^\circ = 0.1736$

**Question 12 (20 Marks)**

**(a) Scale 5B (0, 3, 5)**

*Partial Credit*

- Gives one answer of ( 1 , 0 ) or ( -1 , 0 ) or ( -1, 1 ) or ( 1 , -1 ) .
- Indicates correct point on graph .

**(b)&(c) Scale 15C (0, 4, 8 , 15)**

*Low Partial Credit*

- Part ( b ) correct
- Draws line parallel to given line .

*High Partial Credit*

- Part ( c ) correct

## Bonus marks for answering through Irish

Bonus marks are applied separately to each paper, as follows:

If the mark achieved is 225 or less, the bonus is 5% of the mark obtained, rounded **down**.  
For instance,  $198 \text{ marks} \times 5\% = 9.9 \Rightarrow \text{bonus} = 9 \text{ marks}$ .

If the mark achieved is above 225, the following table applies:

Bunmharc (Mark achieved)	Marc Bónais (Bonus mark)	Bunmharc (Mark achieved)	Marc Bónais (Bonus mark)
226	11	261 – 266	5
227 – 233	10	267 – 273	4
234 – 240	9	274 – 280	3
241 – 246	8	281 – 286	2
247 – 253	7	287 – 293	1
254 – 260	6	294 – 300	0