2014. M329

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014

Mathematics (Project Maths – Phase 3)

Paper 1

Higher Level

Friday 6 June Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

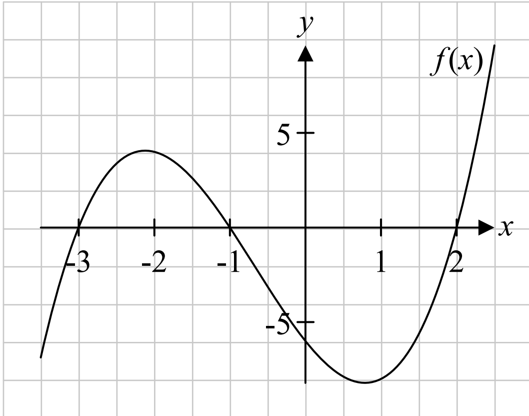
Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) The graph of a cubic function $f(x)$ cuts the x-axis at $x = -3$, $x = -1$ and $x = 2$, and the y-axis at (0, −6), as shown.

Verify that $f(x)$ can be written as

$$f(x) = x^3 + 2x^2 - 5x - 6.$$



(b) **(i)** The graph of the function $g(x) = -2x - 6$ intersects the graph of the function $f(x)$ above. Let $f(x) = g(x)$ and solve the resulting equation to find the co-ordinates of the points where the graphs of $f(x)$ and $g(x)$ intersect.

(ii) Draw the graph of the function $g(x) = -2x - 6$ on the diagram above.

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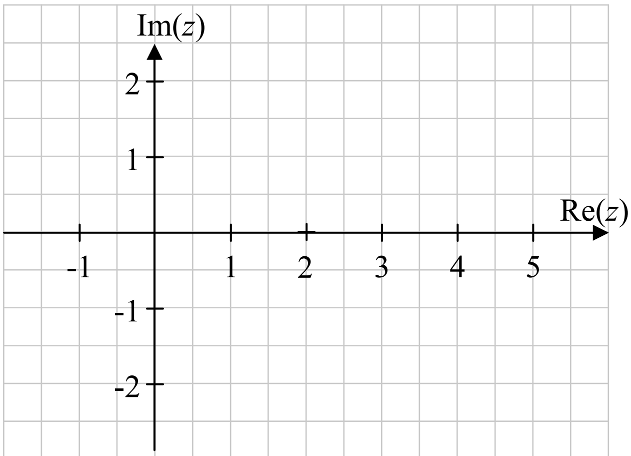
Question 2 **(25 marks)**

Let $z_1 = 1 - 2i$, where $i^2 = -1$.

(a) The complex number z_1 is a root of the equation $2z^3 - 7z^2 + 16z - 15 = 0$.
Find the other two roots of the equation.

(b) **(i)** Let $w = z_1\overline{z_1}$, where $\overline{z_1}$ is the conjugate of z_1. Plot z_1, $\overline{z_1}$ and w on the Argand diagram and label each point.



(ii) Find the measure of the acute angle, $\overline{z_1}wz_1$, formed by joining $\overline{z_1}$ to w to z_1 on the diagram above. Give your answer correct to the nearest degree.

Question 3 **(25 marks)**

(a) Prove, by induction, that the sum of the first n natural numbers,
$1+2+3+\cdots+n$, is $\frac{n(n+1)}{2}$.

(b) Hence, or otherwise, prove that the sum of the first n even natural numbers,
$2+4+6+\cdots+2n$, is n^2+n.

(c) Using the results from **(a)** and **(b)** above, find an expression for the sum of the first n odd natural numbers in its simplest form.

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Question 4 **(25 marks)**

(a) Differentiate the function $2x^2 - 3x - 6$ with respect to x from first principles.

(b) Let $f(x) = \frac{2x}{x+2}$, $x \neq -2$, $x \in \mathbb{R}$. Find the co-ordinates of the points at which the slope of the tangent to the curve $y = f(x)$ is $\frac{1}{4}$.

Question 5 **(25 marks)**

(a) Find $\int 5\cos 3x\, dx$.

(b) The slope of the tangent to a curve $y = f(x)$ at each point (x, y) is $2x - 2$.
The curve cuts the x-axis at $(-2, 0)$.

(i) Find the equation of $f(x)$.

(ii) Find the average value of f over the interval $0 \le x \le 3, x \in \mathbb{R}$.

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Question 6 **(25 marks)**

The n^{th} term of a sequence is $T_n = \ln a^n$, where $a > 0$ and a is a constant.

(a) **(i)** Show that T_1, T_2, and T_3 are in arithmetic sequence.

(ii) Prove that the sequence is arithmetic and find the common difference.

(b) Find the value of a for which $T_1 + T_2 + T_3 + \cdots + T_{98} + T_{99} + T_{100} = 10\,100$.

(c) Verify that, for all values of a,

$$\left(T_1 + T_2 + T_3 + \cdots + T_{10}\right) + 100d = \left(T_{11} + T_{12} + T_{13} + \cdots + T_{20}\right),$$

where d is the common difference of the sequence.

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Section B   Contexts and Applications   150 marks

Answer **all three** questions from this section.

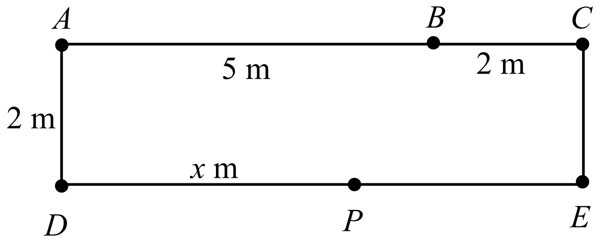
Question 7 **(40 marks)**

(a) Three natural numbers *a*, *b* and *c*, such that $a^2 + b^2 = c^2$, are called a Pythagorean triple.

(i) Let $a = 2n + 1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$.
Pick one natural number *n* and verify that the corresponding values of *a*, *b* and *c* form a Pythagorean triple.

(ii) Prove that $a = 2n + 1$, $b = 2n^2 + 2n$ and $c = 2n^2 + 2n + 1$, where $n \in \mathbb{N}$, will always form a Pythagorean triple.

(b) $ADEC$ is a rectangle with $|AC| = 7$ m and $|AD| = 2$ m, as shown.
B is a point on $[AC]$ such that $|AB| = 5$ m.
P is a point on $[DE]$ such that $|DP| = x$ m.



(i) Let $f(x) = |PA|^2 + |PB|^2 + |PC|^2$.
Show that $f(x) = 3x^2 - 24x + 86$, for $0 \le x \le 7$, $x \in \mathbb{R}$.

(ii) The function $f(x)$ has a minimum value at $x = k$.
Find the value of k and the minimum value of $f(x)$.

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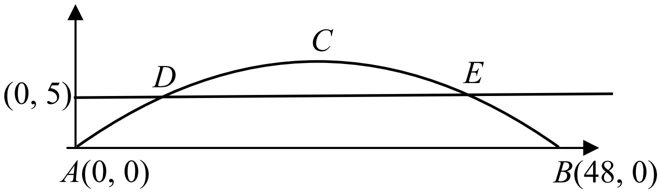
Question 8 **(50 marks)**

In 2011, a new footbridge was opened at Mizen Head, the most south-westerly point of Ireland.

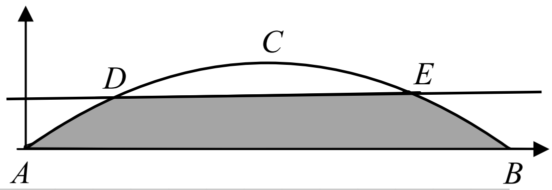
The arch of the bridge is in the shape of a parabola, as shown. The length of the span of the arch, $[AB]$, is 48 metres.



(a) Using the co-ordinate plane, with $A(0, 0)$ and $B(48, 0)$, the equation of the parabola is $y = -0\cdot013x^2 + 0\cdot624x$. Find the co-ordinates of C, the highest point of the arch.

(b) The perpendicular distance between the walking deck, $[DE]$, and $[AB]$ is 5 metres. Find the co-ordinates of D and of E. Give your answers correct to the nearest whole number.

(c) Using integration, find the area of the shaded region, *ABED,* shown in the diagram below. Give your answer correct to the nearest whole number.



(d) Write the equation of the parabola in part **(a)** in the form $y-k=p(x-h)^2$, where k, p, and h are constants.

(e) Using what you learned in part **(d)** above, or otherwise, write down the equation of a parabola for which the coefficient of x^2 is -2 and the co-ordinates of the maximum point are $(3,-4)$.

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Question 9 **(60 marks)**

Ciarán is preparing food for his baby and must use cooled boiled water. The equation $y = Ae^{kt}$ describes how the boiled water cools. In this equation:

- t is the time, in minutes, from when the water boiled,
- y is the *difference* between the water temperature and room temperature at time t, measured in degrees Celsius,
- A and k are constants.

The temperature of the water when it boils is $100°\text{C}$ and the room temperature is a constant $23°\text{C}$.

(a) Write down the value of the temperature difference, y, when the water boils, and find the value of A.

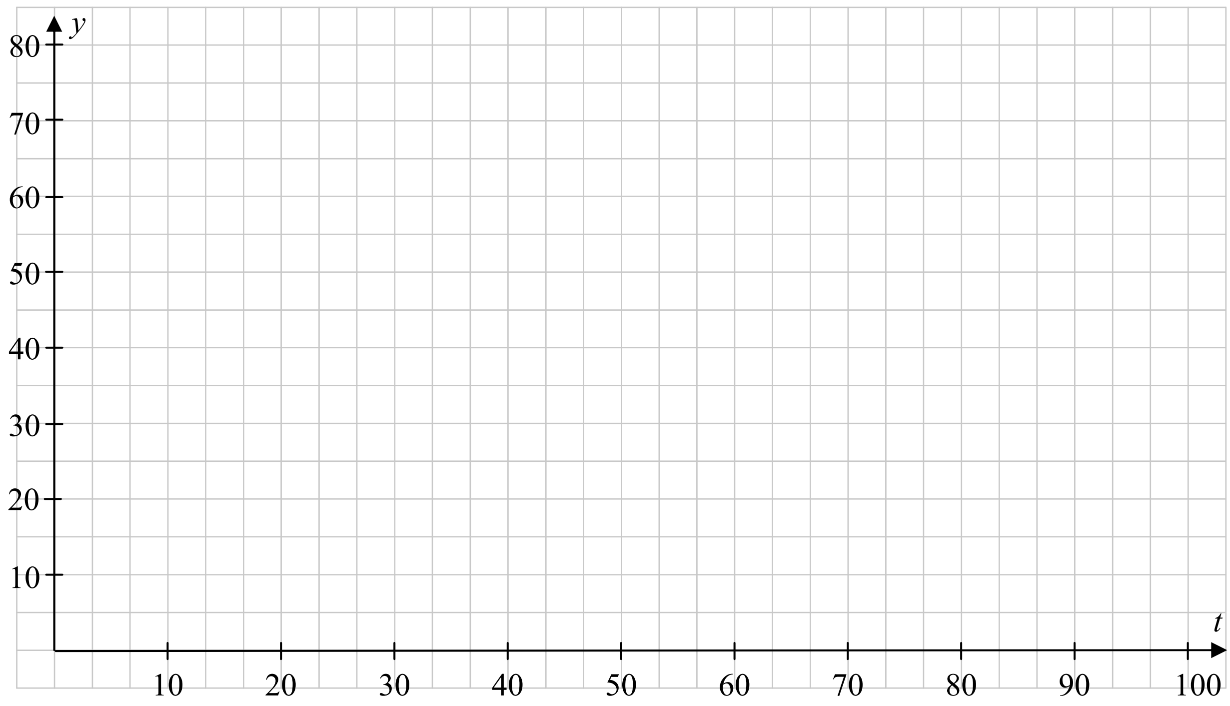
$y =$

$A =$

(b) After five minutes, the temperature of the water is $88°\text{C}$.
Find the value of k, correct to three significant figures.

(c) Ciarán prepares the food for his baby when the water has cooled to $50°\text{C}$. How long does it take, correct to the nearest minute, for the water to cool to this temperature?

(d) Using your values for A and k, sketch the curve $f(t) = Ae^{kt}$ for $0 \le t \le 100,\ t \in \mathbb{R}$.



(e) **(i)** On the same diagram, sketch a curve $g(t) = Ae^{mt}$, showing the water cooling at a *faster* rate, where A is the value from part **(a)**, and m is a constant. Label each graph clearly.

(ii) Suggest one possible value for m for the sketch you have drawn and give a reason for your choice.

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(f) **(i)** Find the rates of change of the function $f(t)$ after 1 minute and after 10 minutes. Give your answers correct to two decimal places.

(ii) Show that the rate of change of $f(t)$ will always increase over time.

You may use this page for extra work.

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