



Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE 2011

MARKING SCHEME

**MATHEMATICS
(PROJECT MATHS)**

ORDINARY LEVEL

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Introduction

The Ordinary Level Mathematics examination for candidates in the 24 initial schools for *Project Maths* shared some content with the examination for all other candidates. The marking scheme used for the shared content was identical for the two groups.

This document contains the complete marking scheme for both papers for the candidates in the 24 schools.

Readers should note that, as with all marking schemes used in the state examinations, the detail required in any answer is determined by the context and the manner in which the question is asked, and by the number of marks assigned to the question or part. Requirements and mark allocations may vary from year to year.



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011

Mathematics
(Project Maths – Phase 2)

Paper 1

Ordinary Level

Friday 10 June Afternoon 2:30 – 4:30

300 marks

Model Solutions – Paper 1

Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	3 questions

Answer questions as follows:

In Section A, answer **all four** questions

In Section B, answer **both** Question 5 **and** Question 6

In Section C, answer **any two** of the three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all four** questions from this section.

Question 1**(25 marks)**

- (a) Explain what a *prime number* is.

A natural number that has 2 factors only

- (b) Express 2652 as a product of prime numbers.

2	2652
2	1326
3	663
13	221
17	17
	1

$$2652 = 2^2 \times 3 \times 13 \times 17$$

- (c) The number $2^{61} - 1$ is a prime number. Using your calculator, or otherwise, express its value, correct to two significant figures, in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in N$.

$$2.3 \times 10^{18}$$

- (d) Use your answer to part (c) to state how many digits there are in the exact value of $2^{61} - 1$.

19 digits

Question 2**(25 marks)**

- (a) A certain deposit account will earn 3% interest in the first year and 6% interest in the second year. The interest is added to the account at the end of each year. If a person invests €20 000 in this account, how much will they have in the account at the end of the two years?

Year 1
 Principal = €20,000
 3 % Interest = € 600

Year 2
 Principal = €20,600
 6 % Interest = € 1,236

Amount at end of 2 years = €21,836

OR

$$20000 \times 1.03 \times 1.06$$

$$= €21,836$$

- (b) Show that, to the nearest euro, the same amount of interest is earned by investing the money for two years in an account that pays compound interest at 4.49% (AER).

$$F = P(1+i)^t \quad P = €20,000 \quad i = 0.0449 \quad t = 2$$

Method 1

$$F = 20000(1+0.0449)^2$$

$$= 20000(1.0449)^2$$

$$F = €21,836.32$$

Rounded to the nearest euro = €21,836

Interest = €1,836

Conclusion: Same amount of interest earned

Method 2

P1	= €20000
4.49%	= €898
P2	= €20898
4.49%	= €938.32

Total Interest = €1836.32

Rounded to nearest euro
 = €1,836

Question 3**(25 marks)**

The terms in an arithmetic sequence are given by the formula

$$T_n = 38 - 4n, \quad \text{for } n = 1, 2, 3, 4, \dots$$

- (a) Write out the first three terms in the sequence.

$$T_1 = 38 - 4(1) = 34$$

$$T_2 = 38 - 4(2) = 30$$

$$T_3 = 38 - 4(3) = 26$$

First 3 three terms are 34, 30, 26

- (b) What is the first negative term in the sequence?

$$T_n < 0 \quad \Rightarrow \quad 38 - 4n < 0$$

$$-4n < -38$$

$$4n > 38$$

$$n > 9.5$$

$$T_{10} \text{ is the first negative term} \quad T_{10} = 38 - 4(10) = -2$$

- (c) Find the sum of the first 15 terms of the sequence.

$$S_n = \frac{n}{2} \{2a + (n-1)d\} \quad n = 15 \quad a = 34 \quad d = -4$$

$$S_{15} = \frac{15}{2} \{2(34) + (15-1)(-4)\}$$
$$= 7.5 \{12\}$$

$$S_{15} = 90$$

- (d) Find the value of n for which the sum of the first n terms of the sequence is 0.

$$S_n = 0 \Rightarrow \frac{n}{2} \{2(34) + (n-1)(-4)\} = 0$$

$$\Rightarrow \frac{n}{2} \{72 - 4n\} = 0$$

$$36n - 2n^2 = 0$$

$$n^2 - 18n = 0$$

$$n(n-18) = 0$$

$$n \neq 0 \quad n = 18 \quad \therefore n = 18$$

Question 4**(25 marks)****(a)** Solve the simultaneous equations:

$$2f + \frac{2}{3}g + 1 = 0$$

$$f + \frac{1}{2}g + 1 = 0.$$

$$6f + 2g = -3$$

$$\underline{2f + g = -2}$$

$$6f + 2g = -3$$

$$\underline{-4f - 2g = 4}$$

$$2f = 1$$

$$\Rightarrow f = \frac{1}{2}$$

$$6\left(\frac{1}{2}\right) + 2g = -3$$

$$3 + 2g = -3$$

$$2g = -6$$

$$g = -3$$

(b) Solve the following inequality, and show the solution set on the number line below.

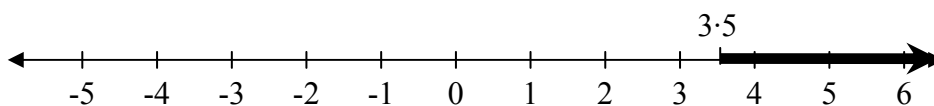
$$5 - \frac{3}{4}x \leq \frac{19}{8}$$

$$40 - 6x \leq 19$$

$$-6x \leq -21$$

$$6x \geq 21$$

$$x \geq 3.5$$

**(Assuming $x \in \mathbf{R}$)**

Answer **both** Question 5 **and** Question 6.

Question 5**(50 marks)**

z is the complex number $1+i$, where $i^2 = -1$.

(a) (i) Find z^2 and z^3 .

$$z^2 = (1+i)(1+i) = 1 + 2i + i^2$$

$$\therefore z^2 = 2i$$

$$z^3 = z^2 z = 2i(1+i) = 2i + 2i^2$$

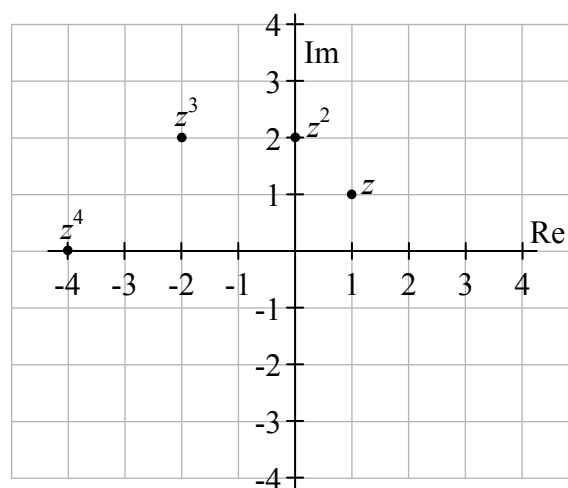
$$\therefore z^3 = -2 + 2i$$

(ii) Verify that $z^4 = -4$.

$$z^4 = z^2 z^2$$

$$\therefore z^4 = (2i)(2i) = 4i^2 = 4(-1) = -4$$

(iii) Show z , z^2 , z^3 and z^4 on the Argand diagram.



(iv) Make one observation about the pattern of points on the diagram.

Getting progressively further from the origin

Or

Moving in an anticlockwise direction

- (b) Using the value of z^4 , or otherwise, find the values of z^8 , z^{12} and z^{16} , and insert their values in the table below.

$$z^8 = (z^4)^2 = (-4)^2 = 16$$

$$z^{12} = (z^4)^3 = (-4)^3 = -64$$

$$z^{16} = (z^4)^4 = (-4)^4 = 256$$

z^4	z^8	z^{12}	z^{16}
- 4	16	- 64	256

- (c) Based on the pattern of values in part (b), or otherwise, state whether z^{40} is positive or negative. Explain how you got your answer.

- z^{40} is positive as all powers that are multiples of 8 are positive.
- Every second term in the table is positive.
 z^{40} is the tenth term in the table so it will be positive.

- (d) Write z^{40} as a power of 2.

Method 1

$$\begin{aligned} z^{40} &= (z^4)^{10} \\ &= (-4)^{10} \\ &= (4)^{10} \\ &= (2^2)^{10} \\ &= 2^{20} \end{aligned}$$

Method 2

$$z^{40} = (-4)^{10} = 1048576 = 2^{20}$$

(e) Find z^{41} .

$$\begin{aligned}z^{41} &= z^{40} z \\ &= 2^{20}(1+i) \\ &= 1048576 + 1048576i\end{aligned}$$

(f) On an Argand diagram, how far from the origin is z^{41} ?

Method 1

$$\begin{aligned}|z^{41}| &= |2^{20}(1+i)| \\ &= \sqrt{(2^{20})^2 + (2^{20})^2} \\ &= \sqrt{2^{40} + 2^{40}} \\ &= \sqrt{2(2^{40})} \\ &= \sqrt{2^{41}} \\ &= 2^{20.5}\end{aligned}$$

Method 2

$$|z^{41}| = |z|^{41} = (\sqrt{2})^{41} = 2^{20} \sqrt{2}$$

Question 6

(50 marks)

At a certain point during the flight of a space shuttle, the booster rockets separate from the shuttle and fall back to earth. The altitude of these booster rockets (their height above sea level) is given by the following formula:

$$h = 45 + \frac{7}{10}t - \frac{1}{200}t^2$$

where h is the altitude in kilometres, and t is the time in seconds after separation from the shuttle.

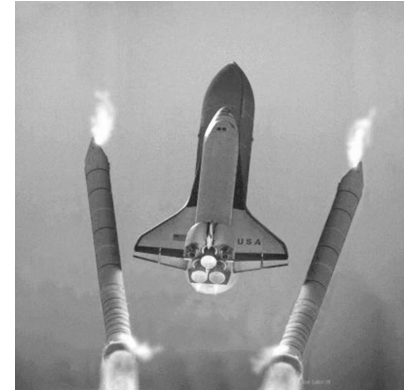


Image: NASA

- (a) Complete the table below, showing the altitude of the rockets at the indicated times.

time in seconds, t	0	20	40	60	80	100
altitude in km, h						

time in seconds, t	0	20	40	60	80	100
altitude in km, h	45	57	65	69	69	65

$$h = 45 + \frac{7}{10}t - \frac{1}{200}t^2$$

Substitute $t = 0, 20, 40, 60, 80$ and 100 into the above formula.

This gives $h = 45, 57, 65, 69, 69$ and 65

- (b) On the opposite page, draw a graph of the altitude of the rockets for the first 100 seconds after separation from the shuttle.

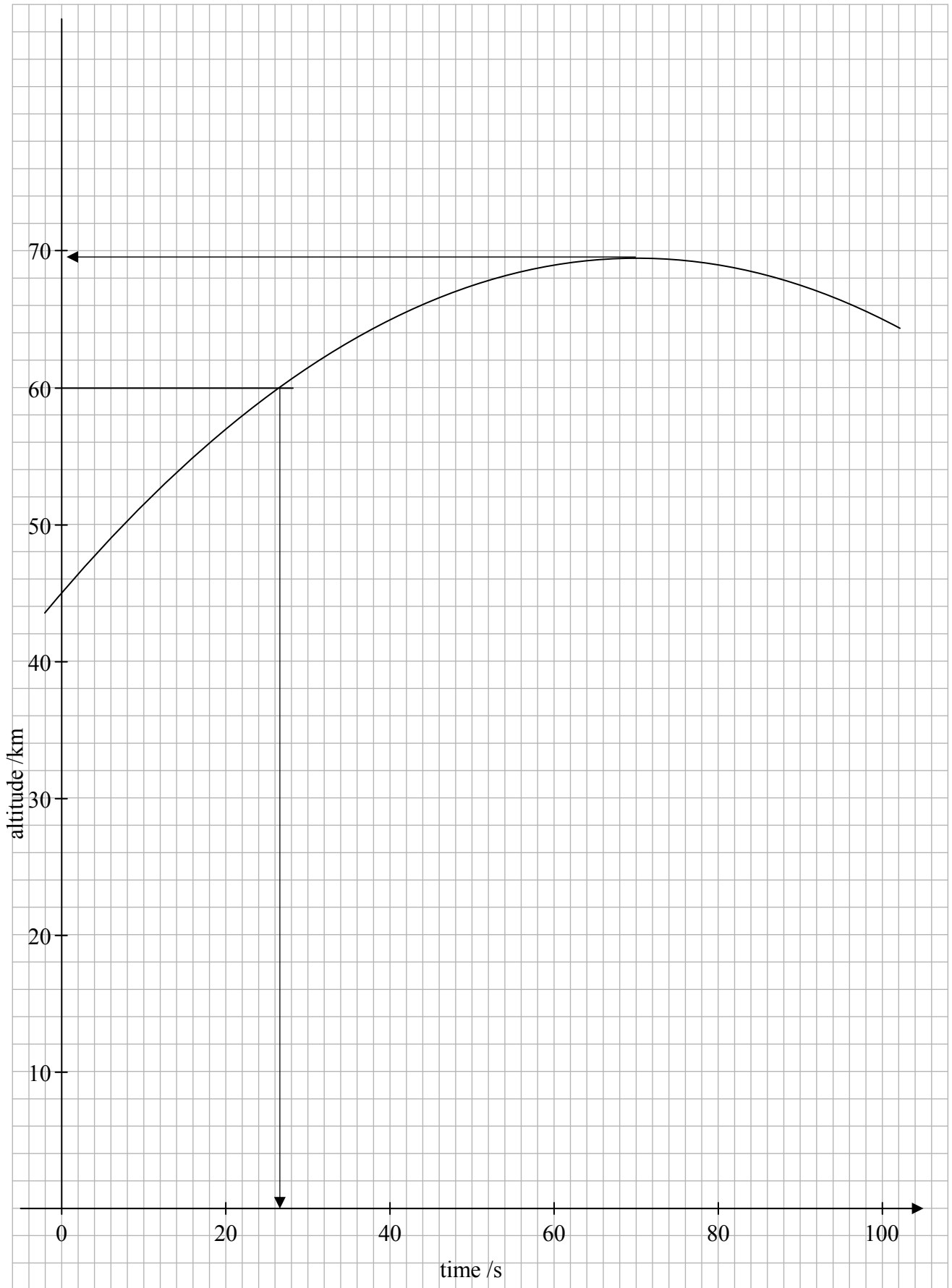
- (c) Use your graph to estimate the greatest altitude reached by the rockets.

Answer: Approximately 69.5 km

- (d) Use the graph to estimate **one** time at which the altitude is 60 km. Show your work clearly on the graph.

Answer: After 26 seconds

Graph of altitude over time.



- (e) Check your answer to part (d) using the formula for the altitude.

$$h = 45 + \frac{7}{10}t - \frac{1}{200}t^2$$
$$\therefore h = 45 + \frac{7}{10}(26) - \frac{1}{200}(26)^2$$
$$h = 59.82 \text{ km}$$

- (f) By solving an equation, find the value of t at which the altitude of the rockets is 9 km.

$$45 + \frac{7}{10}t - \frac{1}{200}t^2 = 9$$
$$9000 + 140t - t^2 = 1800$$
$$t^2 - 140t - 7200 = 0$$
$$(t - 180)(t + 40) = 0$$
$$t = 180 \quad t \neq -40$$

- (g) By finding the change in altitude in one second, or otherwise, find an estimate for the speed at which the rockets are falling when their altitude is 9 km.

$t = 180$ seconds gives an altitude of 9 km

$$t = 181 \Rightarrow h = 45 + \frac{7}{10}(181) - \frac{1}{200}(181)^2 \Rightarrow h = 7.895 \text{ km}$$

$$\therefore 9 - 7.895 = 1.105 \text{ km} \quad \Rightarrow \text{Speed} = 1.105 \text{ km/s} \approx 1.1 \text{ km/s}$$

OR

$$h'(t) = \frac{7}{10} - \frac{t}{100} \Rightarrow h'(180) = -1.1 \Rightarrow \text{Speed} = 1.1 \text{ km/s}$$

Section C**Functions and Calculus (old syllabus)****100 marks**

Answer **any two** of the three questions from this section.

Model solutions for questions 7, 8, and 9 are incorporated into the marking scheme for section C.
See page 27.

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5		
10 mark scale	0, 10	0, 6, 10	0, 5, 8, 10	0, 2, 5, 8, 10	
15 mark scale	0, 15	0, 8, 15	0, 8, 13, 15	0, 7, 10, 13, 15	
20 mark scale	0, 20	0, 10, 20	0, 10, 18, 20	0, 5, 10, 15, 20	
25 mark scale		0, 12, 25	0, 8, 20, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales - level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response almost half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

- (a) 10C
- (b) 5B
- (c) 5B*
- (d) 5B

Question 2

- (a) 15C
- (b) 10C

Question 3

- (a) 10C
- (b) 5C
- (c) 5C
- (d) 5C

Question 4

- (a) 15D
- (b) 10C

Section B

Question 5

- (a) (i) 15C
- (ii) 5B
- (iii) 5B
- (iv) 10A
- (b) 5C
- (c) 5B
- (d), (e) and (f) 5C

Question 6

- (a) 15C
- (b) 10C
- (c) 5A*
- (d) 5B*
- (e) 5B
- (f) 5C
- (g) 5B*

Detailed marking notes

Section A

Question 1

(a) Scale 10C (0, 5, 8, 10)

Low partial credit:

- Writes an example of a prime number (with maximum of one error) in an attempt to list primes

High partial credit:

- Definition of a composite number
- Lists 3 or more correct prime numbers

(b) Scale 5B (0, 3, 5)

Partial credit:

- Writes 2652 as a product
- Some division by a prime number

(c) Scale 5B* (0, 3, 5)

Partial credit:

- Answer not in correct form (see note)
- $2 \cdot 305 \dots$ and stops
- $2^{61-1} = 1 \cdot 1529 \dots \times 10^{18}$

(d) Scale 5B (0, 3, 5)

Partial credit:

- The exponent added to the number of digits in the decimal answer
- Any correct relevant step

Question 2

(a) Scale 15C (0, 8, 13, 15)

Low partial credit:

- Some correct work. For example, interest for one year and stops

High partial credit:

- Correct structure to the answer but with some calculation errors
- $20000(1.06)^2 = \text{€}22,472$ or $20000(1.03)^2 = \text{€}21,218$

(b) Scale 10C (0, 5, 8, 10)

Low partial credit:

- Some correct work.

High partial credit:

- Correct structure to the answer but with some calculation errors

Question 3

(a) Scale 10C (0, 5, 8, 10)

Low partial credit:

- Finds one term only
- Some correct substitution

High partial credit:

- Finds 2 terms correctly
- Correct substitution for all 3 terms but not evaluated

(b) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Insufficient listing, but must list beyond T_3 .
- Some attempt at substitution

High partial credit:

- Stops at $n > 9.5$ or $n = 9.5$
- List method but fails to identify the particular term or number of terms
- Sign error when solving inequality

(c) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Some correct substitution into S_n
- Substitutes correctly for T_{15}

High partial credit:

- Correct substitution into S_n formula but fails to finish
- Lists first 15 terms but incorrect sum

(d) Scale 5C (0, 3, 4, 5)

Low Partial credit:

- Some correct substitution into S_n
- Solves $T_n = 0$ correctly

High partial credit:

- Correct quadratic but fails to solve
- Correct substitution into $S_n = 0$ equation but fails to finish

Question 4

(a) Scale 15D (0, 7, 10, 13, 15)

Low partial credit:

- Some correct attempt at solving
- Some substitution for f and/or g

Middle partial credit:

- Incorrectly solves for one variable and stops
- Attempts to solve for 2 variables but with major errors
- Trial and Error into both equations (once)

High partial credit:

- Solves correctly for one variable only and stops
- Attempts to solve for 2 variables but with minor errors
- Trial and Error into both equations (more than once)

(b) Scale 10C (0, 5, 8, 10)

Low partial credit:

- Some correct attempt at solving or finding a common denominator

High partial credit:

- Solves correctly but incorrect or no number line
- $x \leq 3.5$ and plotted correctly
- Mishandles inequality sign

** Accept $x \in \mathbf{N}, \mathbf{Z}$ or \mathbf{R} when plotting on number line

Section B

Question 5

- (a) (i) Scale 15C (0, 8, 13, 15)

Low partial credit:

- Some correct multiplication or substitution for z

High partial credit:

- Finds z^2 or z^3

- (a) (ii) Scale 5B (0, 3, 5)

Partial credit:

- Writes z^4 as $(1 + i)(1 + i)(1 + i)(1 + i)$ or as $(1 + i)^4$
- Some correct multiplication

- (a) (iii) Scale 5B (0, 3, 5)

Partial credit:

- Any one point plotted correctly (no need to label points)

Note: Accept candidate's answers from (a) (i)

- (a) (iv) Scale 10A (0, 10)

Note: In order to get full credit here, candidates must have plotted at least 2 points in (a) (iii)

- (b) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Correct substitution for z at least once
- Answer given as -4, -8, -12, -16
- Correct work with indices

High partial credit:

- At least one correct value from correct work
- Correct use of indices for all 3 values

- (c) Scale 5B (0, 3, 5)

Partial credit:

- Correct answer but incorrect reasoning
 - Identifies correct pattern but states wrong answer
- * Accept candidate's values from part (b) here

- (d), (e) and (f) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Correct substitution for z with some correct multiplication
- Correct use of indices
- Gives answer to part (d) as 1,048,576
- Modulus formula correctly stated

High partial credit:

- Correct answer to at least one part
- Correct answer to (f) based on candidates incorrect answer to (e) (provided part (e) is not oversimplified)

Question 6

(a) Scale 15C (0, 8, 13, 15)

Low partial credit:

- At least one correct value
- Substitutes correctly at least once

High partial credit:

- At least 4 correct values

(b) Scale 10C (0, 5, 8, 10) (Accept candidates values from (i))

Low partial credit:

- At least one point plotted correctly

High partial credit:

- Points connected with straightedge
- At least 4 points plotted correctly and joined
- Scale error

(c) Scale 5A* (0, 5) (Based on candidate's graph)

Note: Accept answers from 69 km to 72 km for full credit

(d) Scale 5B* (0, 3, 5) (Based on candidate's graph)

Partial credit:

- Correct answer but no work shown on graph
- Uses $t = 60$ to solve
- Outside of tolerance with work shown on graph

Note: Accept answers from 24 seconds to 28 seconds for full credit

(e) Scale 5B (0, 3, 5)

Partial credit:

- Correct substitution of candidate's answer from part (d) into formula

(f) Scale 5C (0, 3, 4, 5)

Low partial credit

- Substitutes $h = 9$ into the formula

High partial credit:

- Correct simplified quadratic equation (or equivalent) but fails to solve, or solves incorrectly.

(g) Scale 5B* (0, 3, 5)

Partial credit:

- Substitutes $t = 181$ or $t = 179$ but fails to finish
- Some correct differentiation
- Speed = $\frac{\text{Distance}}{\text{Time}}$ with substitution.

Marking scheme – Paper 1, Section C

General Guidelines for Examiners – Paper 1, Section C

1. Penalties of three types are applied to candidates' work as follows:

- Blunders - mathematical errors/omissions (-3)
- Slips - numerical errors (-1)
- Misreadings (provided task is not oversimplified) (-1).

Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,..., S1, S2,..., M1, M2,...etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that

- any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
- if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
- a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,...etc.

4. The phrase “hit or miss” means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase “and stops” means that no more work is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

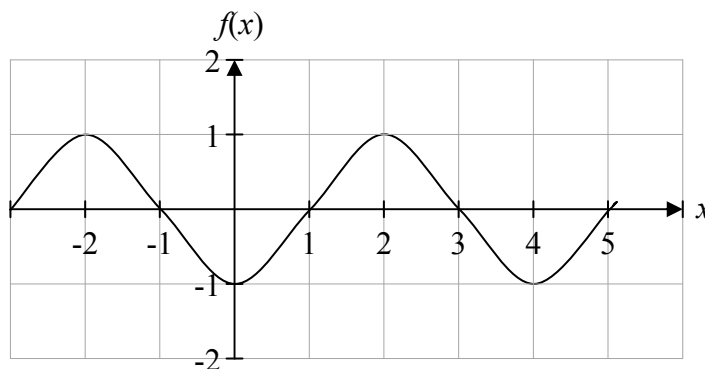
12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 7

Part (a)	15 (10, 5) marks	Att (3, 2)
Part (b)	20 (10, 10) marks	Att (3, 3)
Part (c)	15 (5, 5, 5) marks	Att (2, 2, 2)

Part (a) **15 marks** **Att (3, 2)**

$f : x \rightarrow f(x)$ is a periodic function defined for $x \in \mathbb{R}$.
The period is as indicated in the diagram.



- (i) Write down the period and the range of the function.
(ii) Find $f(71)$.

(a) (i) **10 marks** **Att 3**

(i) Period 4, Range $[-1, 1]$

- * Accept correct answers without work
- * If answers are unidentified assume first is Period, second Range
- * Acceptable notation: Range, $-1 \rightarrow 1$, $[1, -1]$, $-1, 1$ or $(-1, 1)$

Blunders (-3)

- B1 Confuses period and range – once only
- B2 Period = $-2, 2$ or similar
- B3 Range : single number 2
- B4 Only one correct answer

Attempts (3 marks)

- A1 Period and/or range marked on graph but not stated
- A2 No period or worthless period and states range -1 or 1

(a) (ii)

5 marks

Att 2

$$(ii) \quad f(71) = f(68 + 3) = f(3) = 0.$$

[2m] [5m]

- * Accept correct answer without work
- * Accept candidate's value for period from (i)

Blunders(-3)

- B1 Mathematical error e.g. incorrect period
- B2 Incorrect reading (if work shown)

Attempts (2 marks)

- A1 Shows some understanding of period e.g. $f(1) = f(5)$ or similar
- A2 Mentions period, word or figure 4, at this stage and stops e.g. $\frac{71}{4}$ and stops

Part (b)

20 (10, 10) marks

Att (3, 3)

- (i) Differentiate $(4x - 1)(3 - 2x^2)$ with respect to x and simplify your answer.
- (ii) Given that $y = \frac{1}{x^2 - 3x}$, $x \neq 3$, find the range of values of x for which $\frac{dy}{dx} < 0$.

(b) (i)

10 marks

Att 3

I	or	II
$y = (4x - 1)(3 - 2x^2)$		$y = 12x - 8x^3 - 3 + 2x^2$ [3m]
$u = 4x - 1$		
$v = 3 - 2x^2$		
$\frac{du}{dx} = 4$		$\frac{dy}{dx} = 12 - 24x^2 + 4x$ [10m]
$\frac{dv}{dx} = -4x$ [4m]		
$\frac{dy}{dx} = (4x - 1)(-4x) + (3 - 2x^2)(4)$ [7m]		
$= -16x^2 + 4x + 12 - 8x^2$		
$= -24x^2 + 4x + 12$ [10m]		

* Differentiation 7 marks, simplifying 3 marks

* Method II if over simplified when multiplying out only award attempt mark – (must have at least three terms)

* No use of uv , Method I, merits attempt i.e. has $(4)(-4x)$

* Errors in use of uv see guidelines

Blunders (-3)

B1 Differentiation once per term (includes sign)

B2 Differentiation, omitted term

B3 Error when tidying up (final step I) first step II – once only. See comment above for II

Attempts(3 marks)

A1 Some relevant work

(b) (ii)

10 marks

Att 3

I

$$y = \frac{1}{x^2 - 3x}$$

$$u = 1 \quad v = x^2 - 3x$$

$$\frac{du}{dx} = 0 \quad \frac{dv}{dx} = -4x \quad [4m] \quad \text{both required}$$

$$\frac{dy}{dx} = \frac{(x^2 - 3x)(0) - (1)(2x - 3)}{(x^2 - 3x)^2} \quad [7m]$$

$$\frac{dy}{dx} = \frac{-2x + 3}{(x^2 - 3x)^2} < 0 \Rightarrow -2x + 3 < 0 \Rightarrow x > 1.5 \quad [10m]$$

II

$$y = \frac{1}{x^2 - 3x} = (x^2 - 3x)^{-1} \quad [4m]$$

$$\frac{dy}{dx} = -1(2x - 3)(x^2 - 3x)^{-2} \quad \text{or} \quad \frac{-1(2x - 3)}{(x^2 - 3x)^2} \quad [7m]$$

$$\frac{dy}{dx} = \frac{-2x + 3}{(x^2 - 3x)^2} < 0 \Rightarrow -2x + 3 < 0 \Rightarrow x > 1.5 \quad [10m]$$

* Differentiation 7 marks – 3 marks solving $\frac{dy}{dx} < 0$

* No quotient or chain rule in differentiation e.g. has $\frac{dy}{dx}$ as $\frac{0}{2x - 3}$ merits attempt mark only

Blunders (-3)

B1 Differentiation once per term (includes sign)

B2 Differentiation, omitted term

B3 Mathematical error solving inequality – once only

Slips(-1)

S1 Numerical slips

Attempts (3 marks)

A1 Some relevant work e.g. identifies u and /or v and stops

A2 No quotient or chain rule in differentiation e.g. has $\frac{dy}{dx}$ as $\frac{0}{2x - 3}$

A3 Over simplifies to $y = x^2 - 3x$ and continues

Part (c)**15 (5, 5, 5) marks****Att (2, 2, 2)**

Let $f(x) = 2x + \frac{1}{x}$, where $x \in \mathbb{R}$ and $x \neq 0$.

- (i) Find the equation of the tangent to the curve $y = f(x)$ at the point $P(1, 3)$.
 (ii) Q is another point on the curve $y = f(x)$ such that the tangent at Q is parallel to the tangent at P . Find the co-ordinates of Q .

(c) (i)**10 (5, 5) marks****Att (2, 2)**

Step 1 Differentiation: 5 marks

Step 2 Equation of tangent: 5 marks

$$f(x) = 2x + \frac{1}{x} \Rightarrow f'(x) = 2 - \frac{1}{x^2} \quad \text{or} \quad f'(x) = 2 - x^{-2} \quad [5\text{m}]$$

$$\text{or } f(x) = 2x + \frac{1}{x} = \frac{2x^2 + 1}{x} \Rightarrow f'(x) = \frac{x(4x) - (2x^2 + 1)1}{x^2} = \frac{4x^2 - 2x^2 - 1}{x^2} = 2 - \frac{1}{x^2} \quad [5\text{m}]$$

$$f'(x) = 2 - \frac{1}{x^2} \Rightarrow f'(1) = 2 - 1 = 1 \quad [2\text{m}]$$

$$y - 3 = 1(x - 1) \quad [5\text{m}]$$

* $f'(x)$ as $2 + 0/1$ and continues merits at most Att 2 (Step 1) + Att 2 (Step 2)

Blunders (-3)

B1 Differentiation once per term (includes sign) – Step 1

B2 Differentiation, omitted term – Step 1

B3 Mathematical error simplifying $f(x)$

B4 Error finding slope of tangent e.g. use of P – Step 2

Attempts (2 marks)

A1 Simplifies $f(x)$ partially and stops e.g. has $2x + x^{-1}$ Step 1

A2 Some relevant work e.g. states slope of tangent is $f'(x)$

A3 Some effort at finding equation of tangent Step 2 – [Formula of line does not merit attempt mark]

Worthless (0)

W1 Finds $f(1)$ and stops

(c) (ii)**5 marks****Att 2**

$$f'(x) = 2 - \frac{1}{x^2} = 1 \Rightarrow -\frac{1}{x^2} = -1 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1 \quad [2\text{m}]$$

$$f(-1) = 2(-1) + \frac{1}{-1} = -2 - 1 = -3. \quad [5\text{m}] \quad \text{Point } (-1, -3)$$

Blunders (-3)

B1 Mathematical error

Attempts (2 marks)

A1 Some relevant work e.g. sets up $f'(x) = 1$

A2 Finds (1, 3) again

A3 Correct answer without work

QUESTION 8

Part (a)	15 marks	Att 5
Part (b)	20 (10, 10) marks	Att (3, 3)
Part (c)	15 (5, 5, 5) marks	Att (2, 2, 2)

Part (a) **15 marks** **Att5**

Differentiate $x^3 - 7x^2 + 6x$ with respect to x .

(a) **15 marks** **Att 5**

$$\frac{dy}{dx} = 3x^2 - 14x + 6 \quad \text{or} \quad f'(x) = 3x^2 - 14x + 6$$

- * Correct answer without work or notation: full marks
- * If done from first principles, ignore errors in procedure – just mark the answer
- * Only one non zero term correct, award 9 marks

Blunders (-3)

- B1 Differentiation error once per term, (to a maximum of 2) – includes sign
- B2 Term omitted each time

Attempts (5 marks)

- A1 A correct step in differentiation from first principles
- A2 A correct coefficient or a correct index of x in one of the term(s)
- A3 Mentions $\frac{dy}{dx}$ or $f'(x)$

Part (b)

20 (10, 10) marks

Att (3, 3)

- (i) Differentiate $\frac{3x+1}{x-2}$ with respect to x .

Write your answer in the form $\frac{k}{(x-2)^n}$, where $k, n \in \mathbb{Z}$.

- (ii) Given that $y = (x^2 - 2x - 9)^4$, find the value of $\frac{dy}{dx}$ when $x = -2$.

(b) (i)

10 marks

Att 3

$$y = \frac{3x+1}{x-2}$$

$$u = 3x+1 \quad v = x-2$$

$$\frac{du}{dx} = 3 \quad \frac{dv}{dx} = 1 \quad [4m]$$

$$\frac{dy}{dx} = \frac{(x-2)(3) - 1(3x+1)}{(x-2)^2} \quad [9m]$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x-6-3x-1}{(x-2)^2} = \frac{-7}{(x-2)^2} \quad [10m]$$

Note $\frac{dy}{dx} = \frac{-7}{x^2 - 4x + 4} \quad [9m]$

- * Apply penalties as in guidelines
- * No penalty for missing brackets if multiplication implied (decide by later work)
- * No marks for writing u/v formula from tables and stopping
- * No use of u/v formula, has $\frac{dy}{dx}$ as $\frac{3}{1}$ merits attempt mark only

Blunders (-3)

- B1 Differentiation errors, once per term
- B2 Error in formula - see guidelines

Slips (-1)

- S1 Numerical slips

Attempts (3 marks)

- A1 u and/or v correctly identified and stops
- A2 Any correct differentiation

(b) (ii)

10 marks

Att 3

I

$$y = (x^2 - 2x - 9)^4$$

$$\frac{dy}{dx} = 4(x^2 - 2x - 9)^3(2x - 2) \quad [9m]$$

$$x = -2 :$$

$$\frac{dy}{dx} = 4((-2)^2 - 2(-2) - 9)^3(2(-2) - 2) = 24 \quad [10m]$$

II

$$u = (x^2 - 2x - 9)^4 \quad y = u^4$$

$$\frac{du}{dx} = 2x - 2 \quad \frac{dy}{du} = 4u^3 \quad [4m]$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 4u^3(2x - 2) = 4(x^2 - 2x - 9)^3(2x - 2) \quad [9m]$$

$$x = -2 :$$

$$\frac{dy}{dx} = 4((-2)^2 - 2(-2) - 9)^3(2(-2) - 2) = 24 \quad [10m]$$

* Apply penalties as in guidelines for differentiation

* No penalty for missing brackets if multiplication implied (decide by later work)

* Treats $4(x^2 - 2x - 9)^3$ and $(2x - 2)$ as separate parts - see above

* If differentiation correct accept answer **24** with or without work for final marks, answer **24** with no work at all award attempt 3 only

Blunder (-3)

B1 Differentiation error once per part - see parts above e.g. $(2x - 2)$ omitted

Attempts (3 marks)

A1 Some correct element of the chain rule e.g. index 3 or coefficient 4

A2 $u = x^2 - 2x - 9$ and stops

A3 $\frac{dy}{dx} = 2x - 2$ and continues or not, only attempt

Worthless (0)

W1 Substitutes $x = -2$ into y and evaluates y

Part (c)**15 (5, 5, 5) marks****Att (2, 2, 2)**

A ball is rolled in a straight line along a surface.

The distance, s metres, the ball travels is given by

$$s = 18t - 2t^2$$

where t is the time in seconds from the instant the ball begins to move.

- (i) Find the speed of the ball after 3 seconds.
- (ii) How far is the ball from the starting point when it stops moving?
- (iii) Show that the speed of the ball decreases at a constant rate while it is moving.

* Units: Penalise as per guidelines

* No retrospective marking

* No penalty for incorrect notation

* If parts of (c) are unlabelled, and the context doesn't identify which part is which, assume the questions were answered in sequence from (c)(i) to (c)(iii)

(c) (i)**5 marks****Att 2**

$$\frac{ds}{dt} = 18 - 4t = 18 - 4(3) = 6 \text{ m s}^{-1} \text{ at } t = 3$$

* Correct answer without work: Att 2

Blunders (-3)

B1 Mathematical error e.g. differentiation error

B2 Incorrect or no value of t substituted into $\frac{ds}{dt}$

Slips (-1)

S1 Incorrect or no units (only apply if answer correct)

Attempts (2 marks)

A1 Partial differentiation and stops

A2 $\frac{ds}{dt}$ mentioned

Worthless (0)

W1 $t = 3$ substituted into original equation

(c) (ii)

5 marks

Att 2

$$\frac{ds}{dt} = 18 - 4t = 0 \Rightarrow 4t = 18 \Rightarrow t = 4.5 \text{ s}$$
$$s = 18t - 2t^2 = 18(4.5) - 2(4.5)^2 = 40.5 \text{ m}$$

- * No use of derivative merits 0 at this part
- * Accept candidates derivative from (i)

Blunders (-3)

B1 Mathematical error e.g. solving equation

B2 $\frac{ds}{dt} \neq 0$

Slips (-1)

S1 No units or incorrect unit (only apply if answer correct)

Attempts (2 marks)

A1 Some use or mention of derivative at this part

(c) (iii)

5 marks

Att 2

$$\left[\frac{d^2s}{dt^2} \right] = -4 \quad [5\text{m}]$$

- * If candidates 2nd derivative does not give a negative constant apply, slip -1, if explanation not given
- * Candidates may use notation $\frac{dv}{dt}$

Blunders (-3)

B1 Error in differentiation

Attempts (2 marks)

A1 Graphical approach or substitution into $\frac{ds}{dt}$

A2 Mentions acceleration or $\frac{d^2s}{dt^2}$ or $\frac{dv}{dt}$

Worthless (0)

W1 $\frac{ds}{dt}$ and stops

QUESTION 9

Part (i)	15 marks	Att 5
Part (ii)	10 marks	Att 3
Part (iii)	10 marks	Att 3
Part (iv)	5 marks	Att 2
Part (v)	10 marks	Att 3

Part (i) **15 marks** **Att 5**

Let $f(x) = \frac{1}{x+2}$, where $x \in \mathbb{R}$ and $x \neq -2$.

(i) Copy and complete the following table:

x	-5	-4	-3	-2.5	-1.5	-1	0	1
f(x)		-0.5	-1	-2				

(i) **15 marks** **Att 5**

x	-5	-4	-3	-2.5	-1.5	-1	0	1
f(x)	$-\frac{1}{3}$	-0.5	-1	-2	2	1	0.5	$\frac{1}{3}$

* Values of $f(x) = \frac{1}{x+2}$ calculated (all/some correct) misreading which oversimplifies, Att 5

* Accept correct values without work for full marks

* Do not penalise if candidate writes $\frac{1}{3}$ as 0.3

Blunder (-3)

B1 Mathematical error – once if consistent

B2 Treats the function as $f(x) = \frac{1}{x} + 2$ or $f(x) = \frac{1}{x} + \frac{1}{2}$, even if $f(x) = \frac{1}{x+2}$ written

B3 Where no work shown, each missing or incorrect value

Attempts (5 marks)

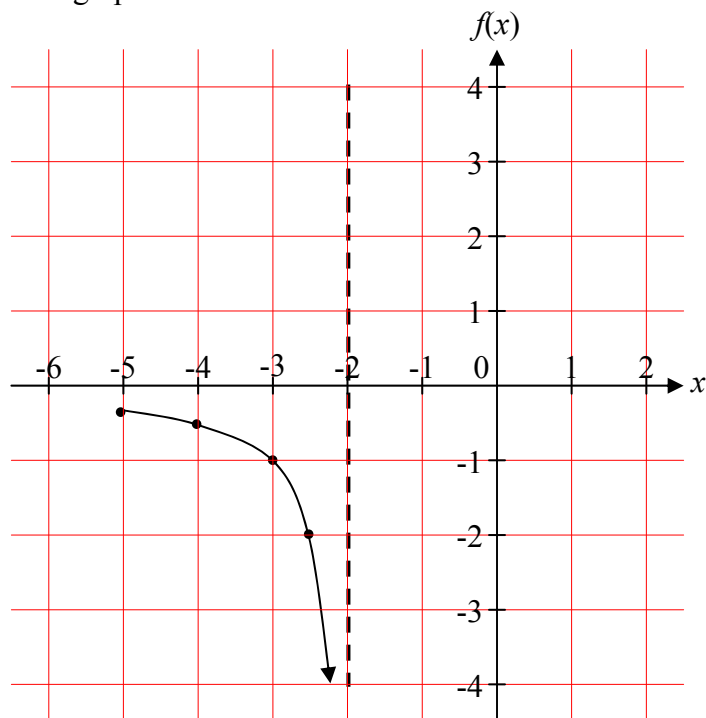
A1 Some relevant work e.g. one non-given value correct

Part (ii)
Part (iii)

10 marks
10 marks

Att 3
Att 3

- (ii) The diagram shows part of the graph of the function f .
Copy and complete the graph from $x = -5$ to $x = 1$.

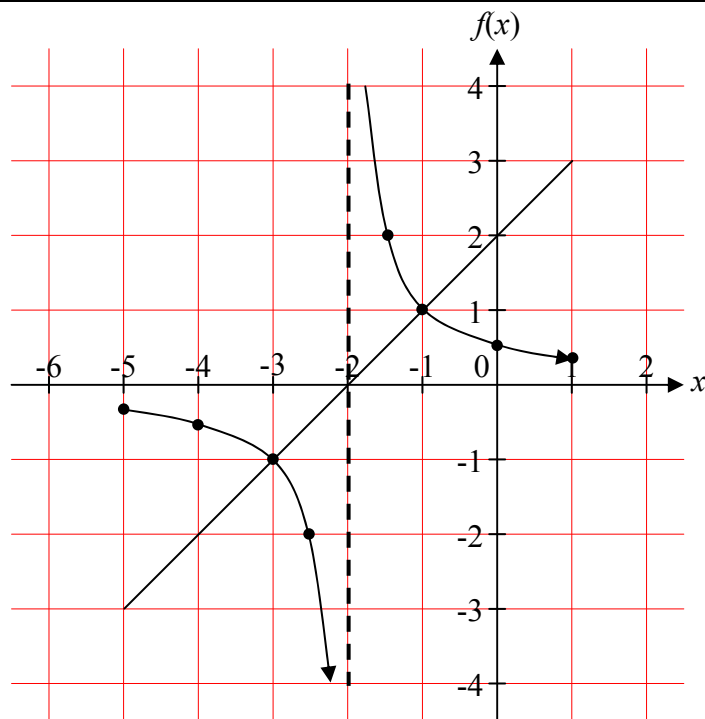


- (iii) On the same diagram, draw the graph of the function $g(x) = x + 2$ in the domain $-5 \leq x \leq 1$, where $x \in \mathbb{R}$.

(ii) $f(x)$
(iii) $g(x)$

10 marks
10 marks

Att 3
Att 3



(ii) $f(x)$ **10 marks****Att 3**

- * Accept candidates values from (i) if not oversimplified
- * If candidates work in section (i) merits 0 marks, award attempt mark at most in section (ii)
- * If candidates re-do without reference to part(i) mark as above – no retrospective marking

Blunders (-3)

- B1 Joins both sides of graph i.e. ignoring asymptote
- B2 Plots points but does not join or joined incorrectly
- B3 Error in plotting once if consistent
- B4 Error in scales

Attempts(3)

- A1 One point correctly plotted and stops

Worthless (0)

- W1 Free hand sketch with no correct points

(iii) $g(x)$ **10 marks****Att 3**

x	-5	-4	-3	-2	-1	0	1
$g(x) = x + 2$	-3	-2	-1	0	1	2	3

[For reference only]

- * Only two points required
- * Two (2) correct points and correct graph award 10 marks
- * Two (2) correct points not resulting in a line graph blunder – 7 marks
- * No penalty for not drawing on same diagram

Blunders(-3)

- B1 Points plotted but not joined
- B2 Incomplete domain

Attempts (3 marks)

- A1 Finds point or plots one correct point

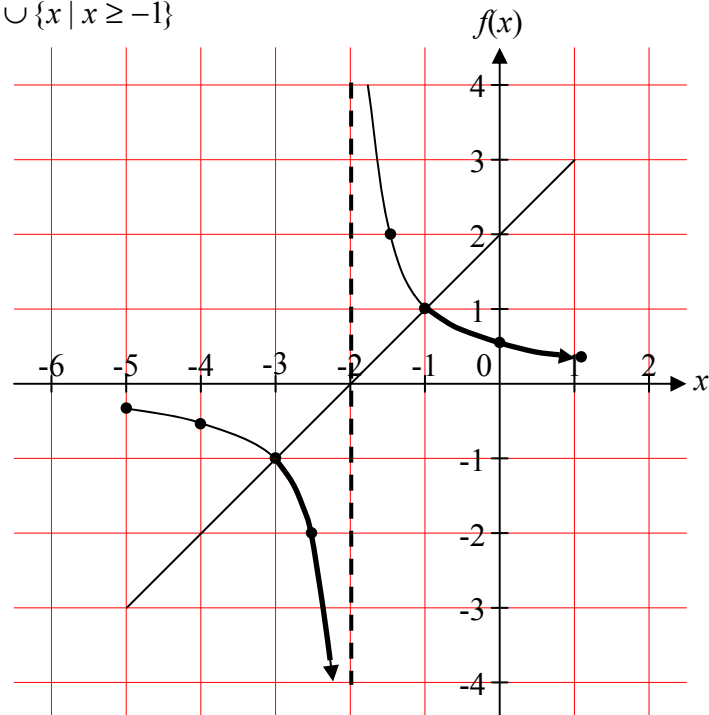
Use your graphs to estimate the range of values of x for which $f(x) \leq g(x)$.

(iv)

5 marks

Att2

$$\{x \mid -3 \leq x < -2\} \cup \{x \mid x \geq -1\}$$



Blunders (-3)

B1 Mathematical error

B2 $f(x) \geq g(x)$

Attempts (2 marks)

A1 Shows on graph only

A2 States/mentions $f(x)$ below $g(x)$

A3 Finds $f(x) \cap g(x)$

Prove that the curve $y = f(x)$ has no turning points.

(v)

10 marks

Att 3

$$f(x) = \frac{1}{(x+2)} = (x+2)^{-1} \quad [4m]$$

$$\Rightarrow f'(x) = -1(x+2)^{-2} = \frac{-1}{(x+2)^2} \quad [7m]$$

$$f'(x) = \frac{-1}{(x+2)^2} = 0 \text{ Impossible or } f'(x) \neq 0 \text{ or } f'(x) < 0 \quad [10m]$$

or

$$f(x) = \frac{1}{(x+2)}$$

$$u = 1, \quad v = x + 2$$

$$\Rightarrow \frac{du}{dx} = 0, \quad \frac{dv}{dx} = 1 \quad [4m]$$

$$f'(x) = \frac{(x+2)(0) - (1)(1)}{(x+2)^2} \quad [7m]$$

$$f'(x) = \frac{-1}{(x+2)^2} = 0 \text{ Impossible or } f'(x) \neq 0 \text{ or } f'(x) < 0 \quad [10m]$$

* Finds $f'(x)$ correctly 7 marks, conclusion 3 marks

* No quotient or chain rule in differentiation, merits attempt mark at most

Blunders (-3)

B1 Differentiation once per term (includes sign)

B2 Differentiation, omitted term

B3 None or incorrect conclusion

Attempts (3 marks)

A1 Mentions $f'(x)$

A2 States function is always “decreasing”



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2011

Mathematics
(Project Maths – Phase 2)

Paper 2

Ordinary Level

Monday 13 June Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note: the model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

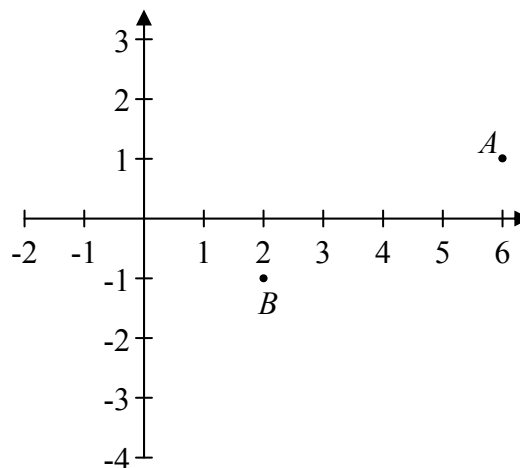
Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1**(25 marks)**

The points $A(6, 1)$ and $B(2, -1)$ are shown on the diagram.



- (a) Find the equation of the line AB .

$$\text{Slope} = \frac{-1-1}{2-6} = \frac{-2}{-4} = \frac{1}{2}$$

Equation:

$$y-1 = \frac{1}{2}(x-6) \quad \text{or} \quad y+1 = \frac{1}{2}(x-2)$$

$$2y-2 = x-6$$

$$x-2y-4 = 0$$

- (b) The line AB crosses the y -axis at C . Find the co-ordinates of C .

$$\text{Cross } y\text{-axis} \Rightarrow x = 0$$

$$\Rightarrow 0 - 2y - 4 = 0$$

$$\Rightarrow -2y = 4$$

$$\therefore y = -2 \quad C(0, -2)$$

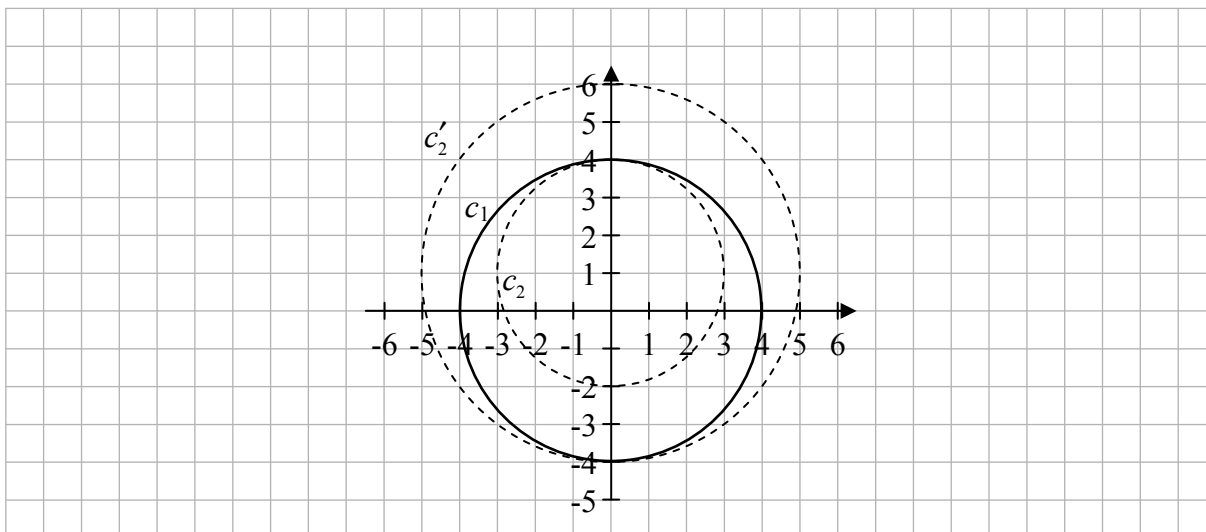
- (c) Find the ratio $\frac{|AB|}{|AC|}$, giving your answer in the form $\frac{p}{q}$, where p and q are whole numbers.

$$\frac{|AB|}{|AC|} = \frac{\sqrt{(2-6)^2 + (-1-1)^2}}{\sqrt{(0-6)^2 + (-2-1)^2}} = \frac{\sqrt{16+4}}{\sqrt{36+9}} = \frac{\sqrt{20}}{\sqrt{45}} = \frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}$$

Question 2**(25 marks)**

A circle c_1 has centre $(0, 0)$ and diameter 8 units.

(a) Show c_1 on a co-ordinate diagram.



(b) Find the equation of c_1 .

$$x^2 + y^2 = (4)^2 \quad [\Rightarrow x^2 + y^2 = 16]$$

(c) Prove that the point $(3, 2)$ is inside c_1 and that the point $(3, 3)$ is outside it.

$$(3, 2): \quad (3)^2 + (2)^2 = 9 + 4 = 13 < 16 \quad \Rightarrow \text{Inside}$$

$$(3, 3): \quad (3)^2 + (3)^2 = 9 + 9 = 18 > 16 \quad \Rightarrow \text{Outside}$$

(d) Another circle, c_2 , has centre $(0, 1)$ and just touches the circle c_1 . Show c_2 on your diagram in part (a) above and find the equation of c_2 .

$$c_2, \text{ has centre } (0, 1) \text{ and radius } 3 \Rightarrow x^2 + (y-1)^2 = 9$$

OR

$$c_2, \text{ has centre } (0, 1) \text{ and radius } 5 \Rightarrow x^2 + (y-1)^2 = 25$$

Question 3**(25 marks)**

A plastic toy is in the shape of a hemisphere. When it falls on the ground, there are two possible outcomes: it can land with the flat side facing down or with the flat side facing up. Two groups of students are trying to find the probability that it will land with the flat side down.

- (a) Explain why, even though there are two outcomes, the answer is not necessarily equal to $\frac{1}{2}$.

- No reason to believe the outcomes are equally likely
- Different shapes of the surface - one side is flat and the rest is curved

- (b) The students estimate the probability by experiment. Group A drops the toy 100 times. From this, they estimate that it lands flat side down with probability 0.76. Group B drops the toy 500 times. From this, they estimate that it lands flat side down with probability 0.812.

- (i) Which group's estimate is likely to be better, and why?

Group B

Reason: In general, the greater the number of trials, the more the estimate tends to the true probability.

- (ii) How many times did the toy land flat side down for Group B?

$$500 \times 0.812 = 406 \text{ times}$$

- (iii) Using the data from the two groups, what is the best estimate of the probability that the toy lands flat side down?

$$100 \times 0.76 = 76 \text{ times}$$

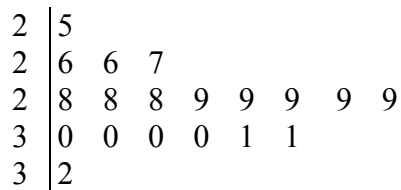
$$\text{Total 'Success' } = 76 + 406 = 482$$

$$\text{Total trials} = 100 + 500 = 600$$

$$\text{Best estimate} = \frac{482}{600} \quad (\approx 0.8)$$

Question 4**(25 marks)**

Below is a stem-and-leaf plot showing the number of sweets in each of nineteen packets of sweets.



Key: 2 | 5 means 25 sweets.

- (a) What is the *median* number of sweets?

Answer: 29 sweets

- (b) What is the *range* of the data?

Range = $32 - 25 = 7$ sweets

- (c) Find the *interquartile range* of the data.

IQR = $Q_3 - Q_1 = 30 - 28 = 2$ sweets

- (d) The sentences below describe the type of data shown in the stem-and-leaf plot above. Delete the incorrect word in each pair of brackets.

“This is a set of [univariate / ~~bivariate~~] data. The data are [discrete / ~~continuous~~].”

Question 5**(25 marks)**

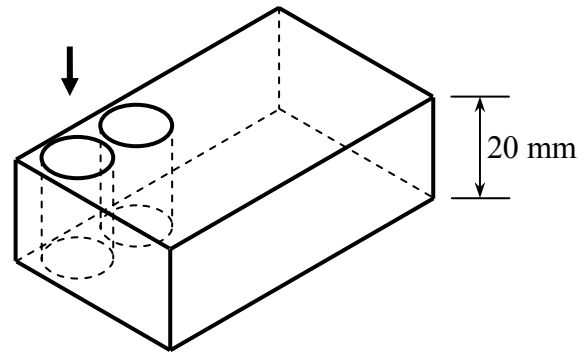
- (a) Find the volume of a cylinder of radius 6 mm and height 20 mm.
Give your answer in two forms, as follows:
- (i) in terms of π , and
 - (ii) correct to two decimal places.

$$(i) \quad V = \pi r^2 h = \pi(6)^2(20) = 720\pi \text{ mm}^3$$

Note: Accept $0.72 \pi \text{ cm}^3$ or $7.2 \times 10^{-7} \pi \text{ m}^3$

$$(ii) \quad V = 720\pi \text{ mm}^3 \approx 2261.94671 = 2261.95 \text{ mm}^3 \quad (2 \text{ decimal places})$$

- (b) A solid rectangular block measures $60 \text{ mm} \times 35 \text{ mm} \times 20 \text{ mm}$.
Cylindrical holes of radius 6 mm are drilled, one at a time, through the block, in the direction shown.
After how many holes will more than half of the original block have been removed?



$$\text{Volume of block} = 60 \times 35 \times 20 = 42,000 \text{ mm}^3$$

$$\text{Volume of one cylindrical hole} = 2261.95 \text{ mm}^3$$

$$\text{Number of holes needed for half the block} = \frac{21,000}{2261.95} \approx 9.28$$

\therefore After 10 holes are drilled, more than half the block will be gone.

Question 6

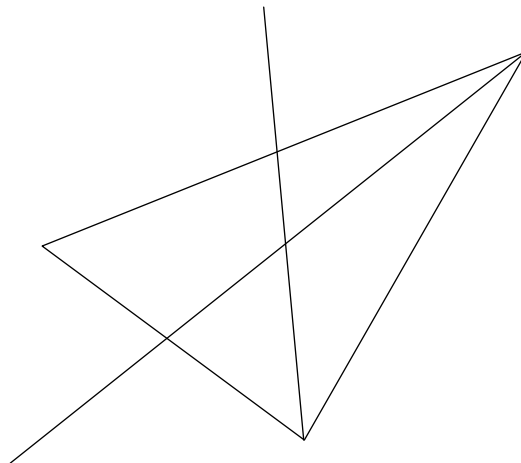
(25 marks)

Answer **either** 6A **or** 6B.

Question 6A

- (a) Show clearly how to construct the centroid of the triangle below.
(Note: all instruments are permitted. If you are using measurements, show your measurements and calculations.)

Centroid: point of intersection of the medians (midpoint to opposite vertex)



- (b) State what is meant by the word *axiom*, and explain why axioms are needed in order to prove theorems.

Axiom: a statement that is accepted without proof

Why needed?

Proofs are based on statements that are already established.

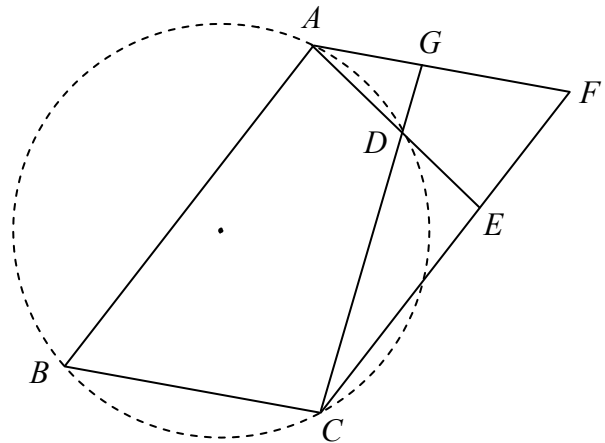
If we had no axioms, we would therefore have no starting point.

OR

Question 6B

In the diagram, $ABCD$ is a cyclic quadrilateral and $ABCF$ is a parallelogram.

Show that $DEFG$ is a cyclic quadrilateral.



$ABCD$ is a cyclic quadrilateral

$$\Rightarrow |\angle ABC| + |\angle ADC| = 180^\circ \quad \text{Step 1}$$

$$\text{But } |\angle ABC| = |\angle AFC| \quad (\text{opposite angle of a parallelogram}) \quad \text{Step 2}$$

$$\text{and } |\angle ADC| = |\angle GDE| \quad (\text{vertically opposite}) \quad \text{Step 3}$$

$\angle AFC$ is the same angle as $\angle GFE$

$$\therefore |\angle GFE| + |\angle GDE| = 180^\circ \quad \text{Step 4}$$

$$\Rightarrow DEFG \text{ is a cyclic quadrilateral} \quad \text{Step 5}$$

Answer Question 7 and Question 8.

Question 7**(75 marks)**

One of the items of information gathered in a census is the *size* of every household. The size of the household is the number of people living in it. The following table shows the number of “Permanent Private Households” of each size in Ireland, according to the census held in various years from 1926 to 2006. For the purposes of this question, you should ignore the fact that there are also other types of household in Ireland.

	1 person	2 people	3 people	4 people	5 people	6 people	7 people	8 people	9 people	≥10 people	All sizes
1926	51,537	98,437	102,664	96,241	82,324	65,310	48,418	33,297	21,089	23,361	622,678
1946	68,881	118,738	116,401	103,423	84,437	62,955	44,028	28,503	17,970	17,318	662,654
1966	88,989	139,541	114,436	97,058	79,320	61,068	42,512	27,098	16,550	20,732	687,304
1986	176,017	195,647	143,142	155,534	127,336	83,657	44,139	23,088	8,438	7,884	964,882
2006	326,134	413,786	264,438	243,303	136,979	54,618	15,141	5,050	1,719	1,128	1,462,296

(Source: Central Statistics Office, <http://www.cso.ie/statistics/HousingandHouseholds.htm>)

(a) Use the information in the table to answer the following:

(i) In 1966, how many households had exactly 8 people living in them?

Answer: 27,098

(ii) In 1986, how many **people** lived in households of exactly 7 people?

$$44,139 \times 7 = 308,973$$

(iii) Calculate, correct to one decimal place, an estimate of the mean number of people per household in 2006.

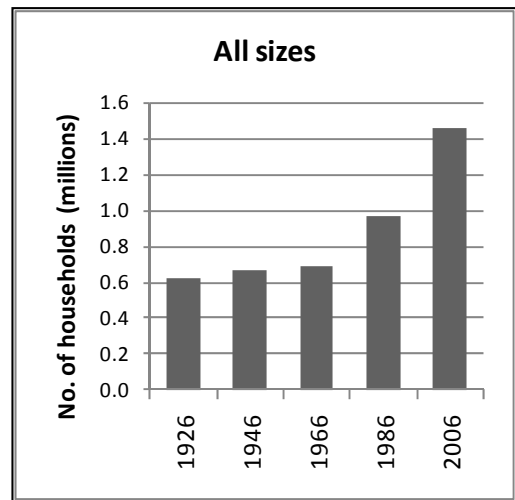
$$\begin{aligned} \text{Mean} &= \frac{(326134 \times 1) + (413786 \times 2) + \dots + (1128 \times 10)}{1462296} \\ &= \frac{4105973}{1462296} \approx 2.8 \text{ people per household} \end{aligned}$$

- (b) Conor, Fiona, and Ray were each asked, separately, to make a presentation about the patterns they could see in the data. They each spoke for one minute and showed one slide. The slides they made are shown below. By considering the slides, state the main point or points that each of them was trying to make.

Conor was trying to show...

- Number of households has more than doubled in the given time period
- The rapid growth from 1966 to 2006

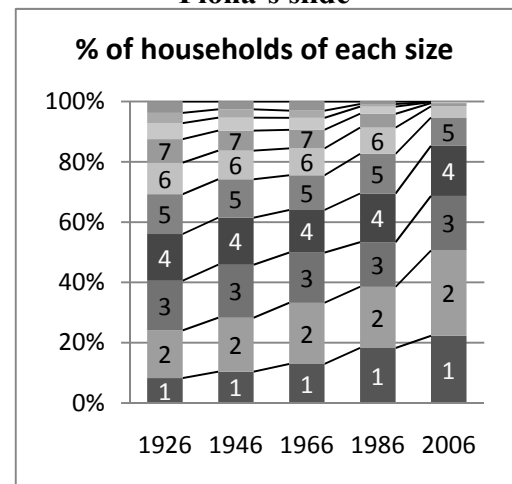
Conor's slide



Fiona was trying to show...

- The gradual reduction in the number of people per household

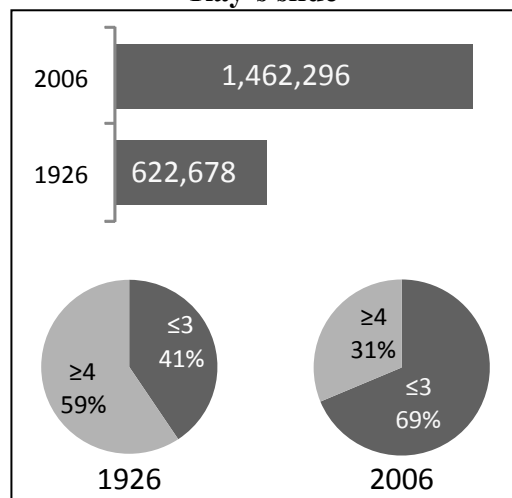
Fiona's slide



Ray was trying to show...

- The number of households has more than doubled in the time period, and the move has been towards smaller household sizes

Ray's slide



- (c) A household is randomly selected from among all the households in 2006. What is the probability that it has seven or eight people?

$$\text{'Success'} = 15,141 + 5050 = 20,191$$

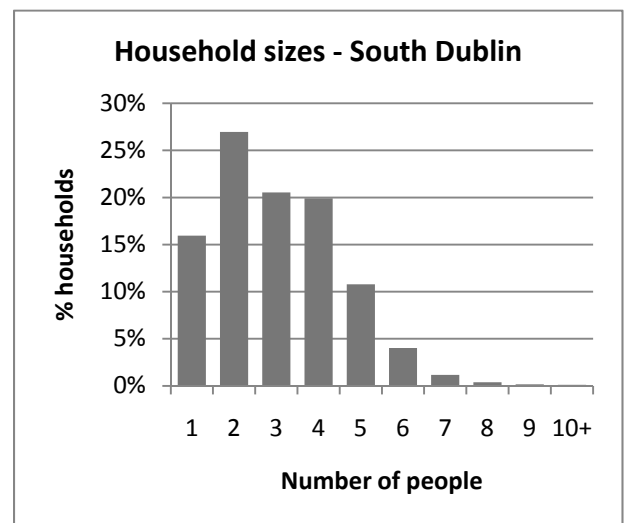
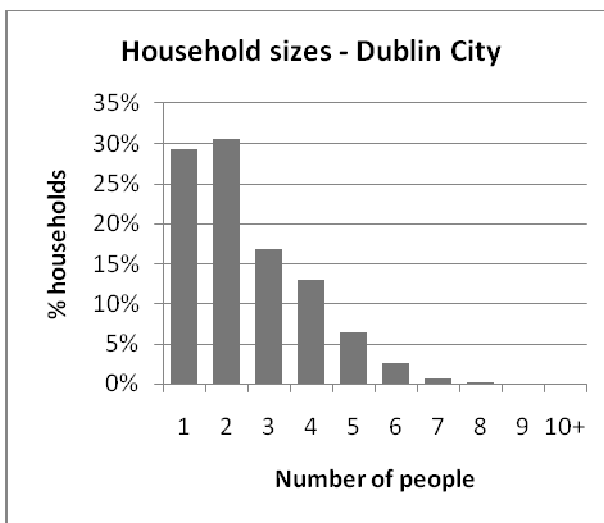
$$\text{Possible} = 1,462,296$$

$$\text{Probability (7 or 8 people)} = \frac{20191}{1462296} (\approx 0.0138)$$

- (d) 1000 households are to be randomly selected from among all the households in 2006. Let X represent the number of 4-person households selected. Find $E(X)$, the expected value of X .

$$E(X) = \frac{243303}{1462296} \times 1000 \approx 166.38$$

- (e) Mary wonders whether there are differences in size between the households in South Dublin and those in Dublin City. She gets the relevant data for 2006 and makes the following charts.



- (i) Describe what differences there are, if any, between the two distributions above.

- In Dublin City there is a much larger percentage of 1-person households.
- Dublin City has a heavy concentration of 1 and 2 person households whereas South Dublin is heavily concentrated in 2, 3 and 4 person households.
- Dublin City has no households of size 9 or 10+

- (ii) There are approximately 81,000 households in South Dublin. Approximately how many people live in 4-person households in South Dublin?

4 persons = 20%
Number of households = $81000 \times 0.2 = 16,200$
Number of People = $16,200 \times 4 = 64,800$

- (iii) What is the median size for a household in Dublin City?

Median Size = 2 people

- (iv) A person is selected at random from among all those living in Dublin City. Which is more likely: that the person lives alone, or that the person lives in a three-person household? Explain your answer.

Live alone (1-person household) $\approx 29\%$

3-person household $\approx 17\%$

But there are 3 people in every 3-person household which means they will outnumber the people living alone.

Therefore it is more likely that the person lives in a three-person household.

Question 8

(75 marks)

The tables in a primary school classroom are like the one in the photograph. The top of the table is in the shape of a trapezium, as shown in the diagram below the photograph.



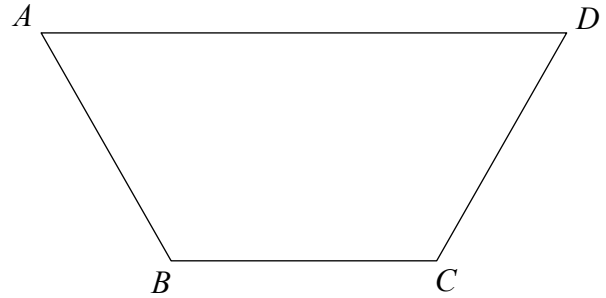
The measurements are as follows:

$$|AD| = 140 \text{ cm}$$

$$|BC| = 70 \text{ cm}$$

$$|AB| = |DC|$$

$$|\angle ADC| = |\angle DAB| = 60^\circ.$$



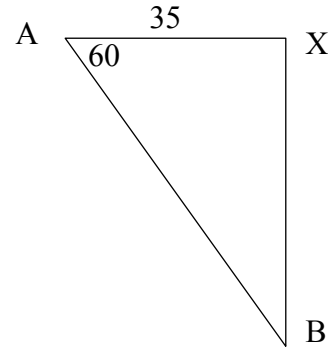
(a) Show that $|AB| = 70 \text{ cm}$.

Triangle ABX is right angled.

$$\cos 60^\circ = \frac{35}{|AB|}$$

$$\Rightarrow \frac{1}{2} = \frac{35}{|AB|}$$

$$\Rightarrow |AB| = 2 \times 35 = 70 \text{ cm}$$



(b) Find the distance between the parallel sides $[AD]$ and $[BC]$. Give your answer in centimetres, correct to one decimal place.

$$\sin 60 = \frac{|BX|}{70} \Rightarrow 70 \sin 60 = |BX| = 70\sqrt{3} = 60.6$$

Distance = 60.6 cm (to 1 decimal place)

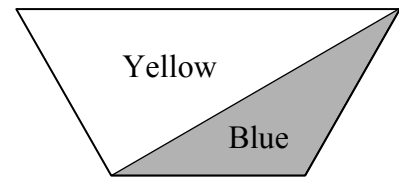
OR

$$|BX|^2 = (70)^2 - (35)^2$$

$$|BX|^2 = 3675$$

$$|BX| = 60.6 \text{ cm}$$

- (c) Some of the tables are painted with a yellow and blue pattern as shown. What fraction of the surface is yellow? Show your work.

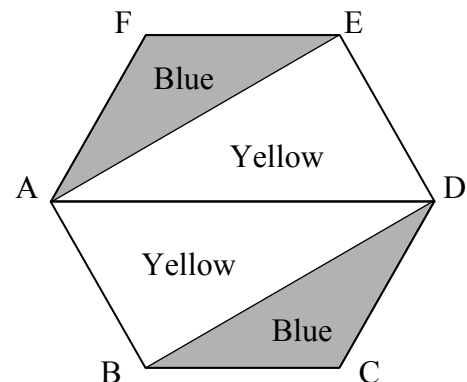


$$\text{Yellow Area} = \frac{1}{2}(140)(60 \cdot 6) = 4,242 \text{ cm}^2$$

$$\text{Blue Area} = \frac{1}{2}(70)(60 \cdot 6) = 2,121 \text{ cm}^2$$

Ratio is 2: 1 Therefore the yellow is $\frac{2}{3}$ of the surface.

- (d) Two of the tables, painted as in part (c) above, are arranged to form a hexagon. Prove that the yellow area is a rectangle.



Opposite sides are clearly equal in length.

We need to show that each corner is 90° .

$$|\angle ABC| = 120^\circ \text{ (regular hexagon)}$$

$$\text{Triangle } BDC \text{ is isosceles} \Rightarrow |\angle CBD| = \frac{1}{2}(180^\circ - 120^\circ) = 30^\circ$$

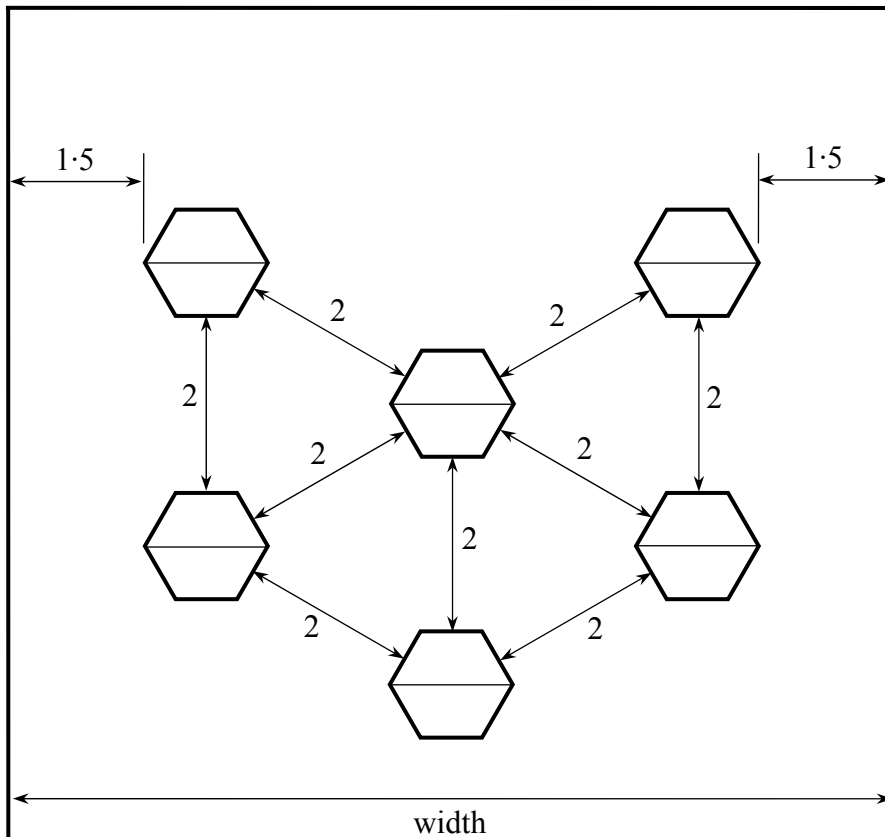
$$\therefore |\angle ABD| = 120^\circ - 30^\circ = 90^\circ$$

Similarly, for the other 3 angles in the yellow area.

Therefore, the yellow area is a rectangle.

- (e) Twelve of the tables are arranged as six hexagons in a classroom, as shown in the diagram. The clearance between neighbouring tables is 2 metres and the clearance to the side walls is 1.5 metres, as shown.

Find the total width of the classroom, in metres, correct to two decimal places.



We can form an equilateral triangle of side $[2 + 2(0.606)]$ metres

Side length = 3.212 m

$$\text{Height of triangle} = \sqrt{(3.212)^2 - (1.606)^2}$$

Height ≈ 2.78 m

$$\begin{aligned} \text{Total Width} &= 2[1.5 + 0.7 + 2.78] \\ &= 2 [4.98] \\ &= 9.96 \text{ metres} \end{aligned}$$

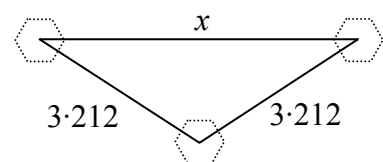
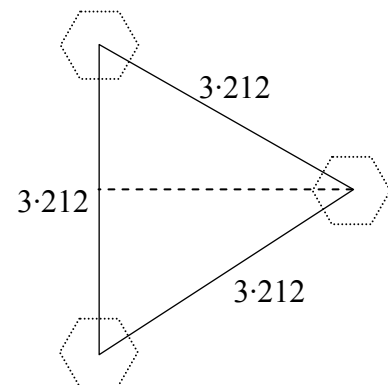
OR

$$x^2 = 3.212^2 + 3.212^2 - 2(3.212)(3.212)\cos 120^\circ$$

$$x^2 = 30.9508$$

$$x = 5.5633$$

$$\text{Total Width} = 5.5633 + 2(0.7) + 2(1.5) = 9.96 \text{ metres}$$



- (f) The tops of the trapezium tables are made of wood. The wood is 1.6 cm thick. Each cubic centimetre of the wood weighs 0.75 grams. Each table also has a metal frame weighing 6 kilograms. How much does each table weigh? Give your answer in kilograms, correct to one decimal place.

$$\text{Area Trapezium} = \left(\frac{a+b}{2} \right) h = \frac{(140+70)}{2} (60.6) = 6363 \text{ cm}^2$$

$$\begin{aligned} \text{Volume of wood needed} &= \text{area} \times \text{thickness} \\ &= 6363 \times 1.6 = 10,180.8 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Weight of wood} &= \text{volume} \times 0.75 \\ &= 10180.8 \times 0.75 = 7635.6 \text{ grams} \end{aligned}$$

$$\text{Weight of wood} = 7.6356 \text{ kg}$$

$$\text{Weight of frame} = 6 \text{ kg}$$

$$\text{Total weight of table} = 13.6356 \text{ kg} = 13.6 \text{ kg (correct to one decimal place)}$$

Marking Scheme – Paper 2

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5		
10 mark scale	0, 10	0, 6, 10	0, 5, 8, 10	0, 2, 5, 8, 10	
15 mark scale	0, 15	0, 8, 15	0, 8, 13, 15	0, 7, 10, 13, 15	
20 mark scale	0, 20	0, 10, 20	0, 10, 18, 20	0, 5, 10, 15, 20	
25 mark scale		0, 12, 25	0, 8, 20, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response almost half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

In certain cases, typically involving incorrect rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, *scale 10C** indicates that 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1

- (a) 15D
- (b) 5C
- (c) 5C

Question 2

- (a) 10C
- (b) 5B
- (c) 5C
- (d) 5C

Question 3

- (a) 5B
- (b) (i) 10C
- (b) (ii) 5B
- (b) (iii) 5B

Question 4

- (a) 10B
- (b) 5C
- (c) 5C
- (d) 5B

Question 5

- (a) (i) 10C*
- (a) (ii) 5B*
- (b) 10C

Question 6A

- (a) 15C
- (b) 10C

Question 6B

25C

Section B

Question 7

- (a)(i) 15B
- (a) (ii) 5B
- (a) (iii) 5C*
- (b) 5C
- (c) 10C
- (d) 5C
- (e) (i) 15B
- (e) (ii) 5C
- (e) (iii) 5B
- (e) (iv) 5C

Question 8

- (a) 15C
- (b) 20C*
- (c) 20C
- (d) 5C
- (e) 10C*
- (f) 5C*

Detailed marking notes

Section A

Question 1

(a) Scale 15D (0, 7, 10, 13, 15)

Low partial credit:

- Any correct step

Middle partial credit:

- Error in finding slope and equation of line

High partial credit:

- Error in finding slope or equation of line

(b) Scale 5C (0, 3, 4, 5)

Low Partial credit:

- Puts $x = 0$ and stops
- Puts $y = 0$ and continues
- Reads (4, 0) from graph

High Partial credit:

- Error(s) in solving for y
- Reading correct answer from graph
- Correct answer with no work

(c) Scale 5C (0, 3, 4, 5)

Low Partial credit:

- Some correct substitution into relevant formula

High Partial credit:

- Answer not in correct form
- One distance found correctly

Question 2

(a) Scale 10C (0, 5, 8, 10)

Low partial credit:

- Centre and diameter incorrect
- Any attempt at drawing a circle

High partial credit:

- Centre or diameter incorrect

(b) Scale 5B (0, 3, 5)

Partial credit:

- Incorrect substitution into relevant formula

Note: Allow radius used in part (a) for full credit

(c) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Shows at least one point correctly on diagram
- Substitutes into relevant formula

High partial credit:

- Proves for one point only
- Substitutes both points correctly but no/incorrect conclusion

(d) Scale 5 (0, 3, 4, 5)

Low partial credit:

- Drawing another circle with centre (0, 1)
- Substitutes into relevant formula

High partial credit:

- Correct diagram or correct equation

Question 3

(a) Scale 5B (0, 3, 5)

Partial credit:

- Attempts to justify that the answer is $\frac{1}{2}$

(b) (i) Scale 10C (0, 5, 8, 10)

Low partial credit:

- States Group A with attempt to justify

High partial credit:

- States Group B but incorrect reason

(ii) Scale 5B (0, 3, 5)

Partial credit:

- Some use of 500 or 0·812

(iii) Scale 5B (0, 3, 5)

Partial credit:

- Adds 100 to 500
- Some use of 0·76 and 0·812
- Repeats answer from (b) (i)
- Correct answer without work
- Writes 76

Question 4

(a) Scale 10B (0, 6, 10)

Partial credit:

- Writes 28·789 (Mean)
- Writes 2·9 or 290
- Writes 9 (omits stem)

(b) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Mentions 32 or 25
- Verbal description of range
- Writes 5 and 2

High Partial credit:

- Writes as 32 to 25
- $3 \cdot 2 - 2 \cdot 5 = 0 \cdot 7$

(c) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Finds Q_3 or Q_1
- Verbal description of interquartile range
- Some use of $\frac{1}{4}$ or $\frac{3}{4}$
- Writes 8 and 0

High Partial credit:

- Finds Q_3 and Q_1 but fails to subtract

(d) Scale 5B (0, 3, 5)

Partial credit:

- One correct deletion

Question 5

(a) (i) Scale 10C* (0, 5, 8, 10)

Low partial credit:

- Some correct substitution into a relevant formula

High partial credit:

- Correct substitution for r and h

(a) (ii) Scale 5B* (0, 3, 5)

Partial credit:

- Some correct substitution into a relevant formula

(b) Scale 10C (0, 5, 8, 10)

Low partial credit:

- Finds volume of block or uses answer from part (a)
- Some correct substitution into a relevant formula
- Attempt at counting number of cylinders by comparing diameter with dimensions of box

High partial credit:

- Correct volume of block and volume of cylinder
[allow candidate's answer from (a)]
- Correct structure to question but with arithmetic errors

Question 6A

(a) Scale 15C (0, 8, 13, 15)

Low partial credit:

- Any valid attempt at construction of any centre of the triangle

High partial credit:

- Constructs incentre, circumcentre or orthocentre
- Constructs one median correctly

Note: Allow tolerance of ± 3 mm when finding midpoints

(b) Scale 10C (0, 5, 8, 10)

Low partial credit:

- Incorrect definition and justification of partial merit

High partial credit:

- Correct definition but incorrect justification
- Correct justification but incorrect definition

Question 6B

Scale 25C (0, 8, 20, 25)

Low partial credit:

- At least one correct step

High partial credit:

- One or two steps missing or incorrect

Section B

Question 7

- (a) (i) Scale 15B (0, 8, 15)

Partial credit:

- Incorrect figure from column '8 people' or row '1966'

- (a) (ii) Scale 5B (0, 3, 5)

Partial credit:

- 44,139 and stops
- Incorrect figure from column '7 people' multiplied by 7

- (a) (iii) Scale 5C* (0, 3, 4, 5)

Note: Accept answer between 2.6 and 3.0, with work, for full credit

Exception: Accept 2.8 with no work for full credit

Low partial credit:

- Any correct relevant step

High partial credit:

- Answer given between 2.6 and 3.0 but no work shown
- Formula for mean with correct substitution
- Correct numerator or denominator in fraction answer

- (b) Scale 5C* (0, 3, 4, 5)

Low partial credit:

- One correct observation

High partial credit:

- 2 correct observations

- (c) Scale 10C (0, 5, 8, 10)

Note: Accept correct answer with no work for full credit

Low Partial credit:

- Writes 15141 or 5050

High partial credit:

- Correct numerator or correct denominator
- Correct probability for a different year (for 7 or 8 people)

- (d) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Correct numerator or denominator
- Multiplies by 1000
- Correct expected value for another year (for 4-person household)

High partial credit:

- $\frac{243303}{1462296}$ and stops

(e) (i) Scale 15B (0, 8, 15)

Partial credit:

- A relevant statement
- States a similarity between the two distributions

(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Multiplies incorrect number of households by 0.2
- Mentions 20%
- 81000×0.12 (or 0.13) and stops

High partial credit:

- 16200 and stops
- 81000×0.12 (or 0.13) and continues

(iii) Scale 5B (0, 3, 5)

Partial credit:

- Median = 3
- Finds the middle number of any relevant set of numbers

(iv) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Uses South Dublin data
- Mentions 29% or 17% (or similar)

High partial credit:

- States living alone by comparing both percentages
- Correct answer without explanation

Question 8

(a) Scale 15C (0, 8, 13, 15)

Low partial credit:

- Any correct relevant step
- Relevant addition to the diagram

High partial credit:

- Correct substitution into trigonometric ratio
- Correct method but with errors
- Assumes triangle ABC or triangle BCD is isosceles (with some work shown)

(b) Scale 20C* (0, 10, 18, 20)

Low partial credit:

- Any correct relevant step
- Relevant addition to the diagram

High partial credit:

- Correct substitution into trigonometric ratio
- Correct method but with errors

(c) Scale 20C (0, 10, 18, 20)

Low partial credit:

- Any correct relevant step
- Correct relevant formula
- Relevant addition to the diagram
- Any estimate greater than $\frac{1}{2}$ and less than 1

High partial credit:

- Area of yellow and/or blue triangle found correctly
- Correct answer without work
- Correct area for trapezium
- Gives answer as $\frac{1}{3}$ with or without work
- Correct method but with errors

(d) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Any correct relevant statement

High partial credit:

- States opposite sides are equal in length or proves one angle is 90°

(e) Scale 10C* (0, 5, 8, 10)

Low partial credit:

- Use of scale diagram
- Any correct relevant statement (e.g. $1.5 + 1.5$)
- Relevant addition to the diagram

High partial credit:

- Correct method but with errors

(f) Scale 5C* (0, 3, 4, 5)

Low partial credit:

- Any correct relevant step
- Correct relevant formula

High partial credit:

- Weight of wood correct with subsequent errors
(Accept candidate's answer from part (b))

Marcanna Breise as ucht Freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. $198 \text{ marc} \times 5\% = 9.9 \Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle $[300 - \text{bunmharc}] \times 15\%$, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

