Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination, 2012

Mathematics
(Project Maths – Phase 3)

Paper 1

Ordinary Level

Friday 8 June   Afternoon 2:00 – 4:30

300 marks

Examination number

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Centre stamp

Running total	

Grade

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1   **(25 marks)**

Alan pays income tax, a universal social charge (USC) and pay-related social insurance (PRSI) on his gross wages. His gross weekly wages are €510.

(a)   Alan pays income tax at the rate of 20%. He has weekly tax credits of €63. How much income tax does he pay?

(b)   Alan pays the USC at the rate of 2% on the first €193, 4% on the next €115 and 7% on the balance. Calculate the amount of USC Alan pays.

(c)   Alan also pays PRSI. His total weekly deductions amount to €76·92. How much PRSI does Alan pay?

page   running

Question 2 (25 marks)

Let $a = \sqrt{2}$.

(a) For each of the numbers in the table below, tick ($\checkmark$) the correct box to say whether it is *rational* or *irrational*.

Number	rational	irrational
a		
$a - 1$		
$(-a)^2$		
$(a-2)^2$		
$1 + a^2$		

(b) Show the following numbers on the number line below.

$$a, \qquad -a, \qquad \sqrt{a}, \qquad a^{-2}$$
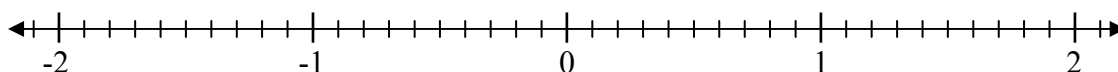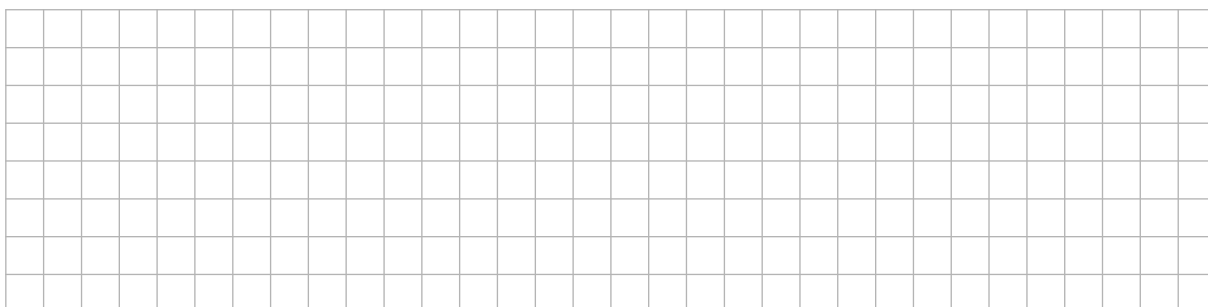
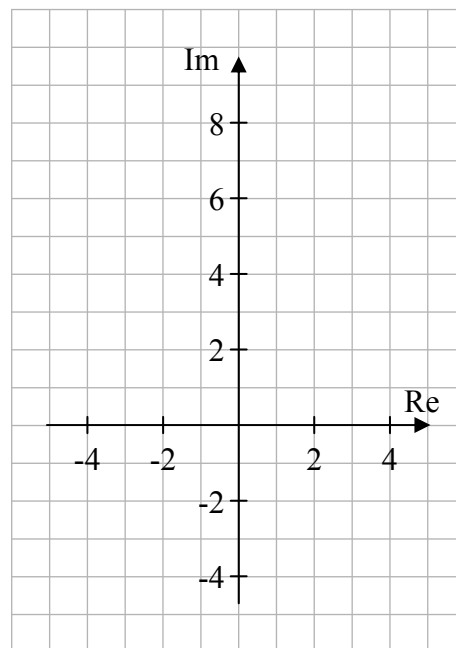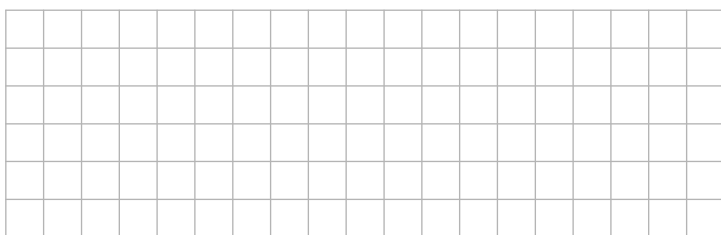**(c)** Verify that $3 - \sqrt{2}$ is a root (solution) of the equation $x^2 - 6x + 7 = 0$.

Question 3 (25 marks)
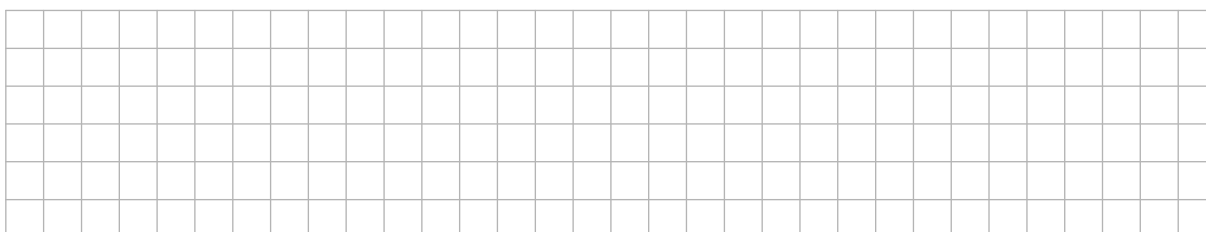
The complex number $z = 1 - 4i$, where $i^2 = -1$.

(a) Plot z and $-2z$ on the Argand diagram.

(b) Show that $2\,|\,z\,| = |\,{-2z}\,|$.

(c) What does part **(b)** tell you about the points you plotted in part **(a)**?

(d) Let k be a real number such that $|\,z + k\,| = 5$. Find the two possible values of k.
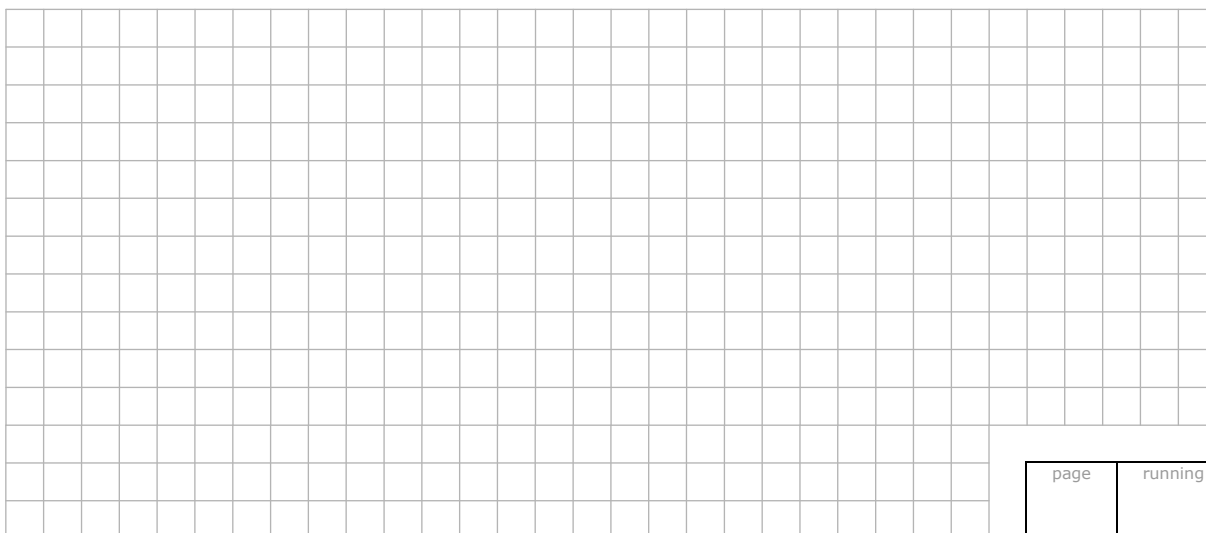
Question 4 (25 marks)

(a) Solve the equation $\frac{1}{2}(7x-2)+5=2x+7$.

(b) Solve the equation $\dfrac{2}{3x-4}-\dfrac{1}{2x+1}=\dfrac{1}{2}$ and give your answers correct to one decimal place.
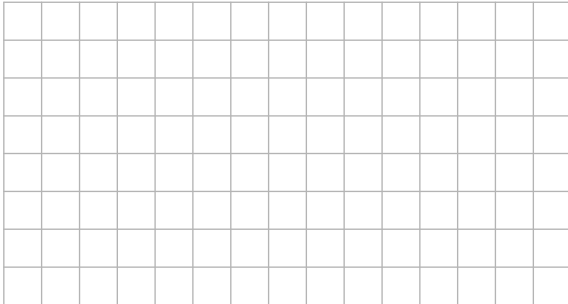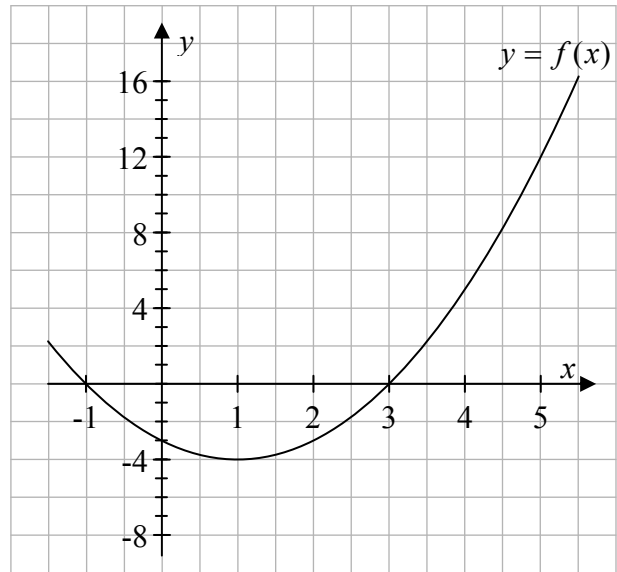
Question 5   (25 marks)

The diagram shows the graph of a function f.

(a) The graph of another function g is a straight line.

$g(-1) = -6$ and $g(3) = 6$.

Draw the graph of g on the diagram.

(b) Use the graphs to find the two values of x for which $g(x) = f(x)$.
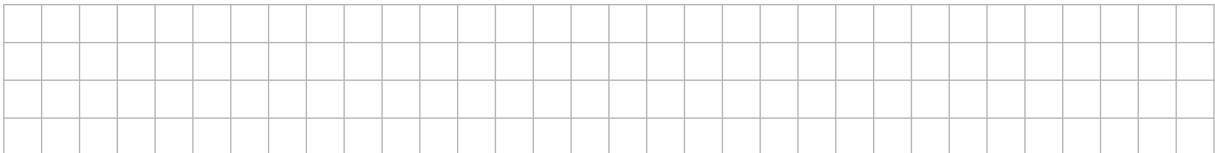
(c) The functions g and f are defined for $x \in \mathbb{R}$ by:

$$g : x \mapsto ax + b$$
$$f : x \mapsto x^2 + px + q$$

where a, b, p, and q are constants.
The graph of f crosses the x-axis at -1 and 3, as shown.
By finding the values of a, b, p, and q, use algebra to solve $g(x) = f(x)$.
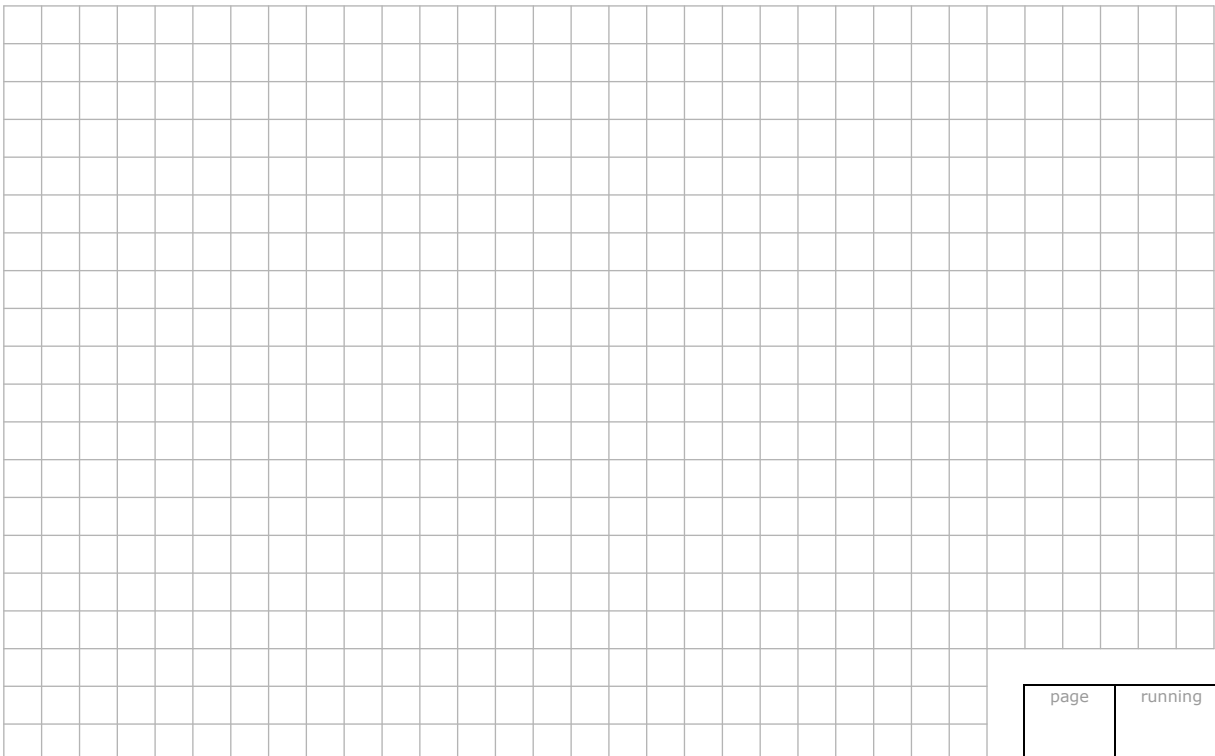
page	running

Question 6 (25 marks)

The diagram shows the graph of the cubic function f,
defined for $x \in \mathbb{R}$ as

$$f : x \mapsto x^3 - x^2 - x + 6.$$

(a) Find the co-ordinates of the point at which f cuts the y-axis.

(b) f has a minimum turning point at $(1, 5)$.
Find the co-ordinates of the maximum turning point.

(c) The lines k and l are tangents to the curve $y = f(x)$ and l is parallel to k. The equation of k is $4x - y + 9 = 0$. Find the x co-ordinate of the point at which l is a tangent to the curve.

Answer **all three** questions from this section.

Question 7   **(50 marks)**

Doctors sometimes need to work out how much medicine to give a child, based on the correct dose for an adult. There are different ways of doing this, based on the child's age, weight, height, or some other measure.

(a) One rule for working out the child's dose from the adult dose is called *Clark's rule*. It is:

$$C = \left(\frac{W}{68}\right) \times A$$

where C is the child's dose, A is the adult's dose, and W is the child's weight in kilograms.

The adult dose of a certain medicine is 125 mg per day. Calculate the correct dose for a child weighing 30 kg, using Clark's rule. Give the answer correct to the nearest 5 mg.

(b) Another rule for working out the child's dose is called *Young's rule*. Below are three different descriptions of Young's rule, taken from the internet. In each case, write down a formula that *exactly* matches the description in words. State clearly the meaning of any letters you use in your formulae.
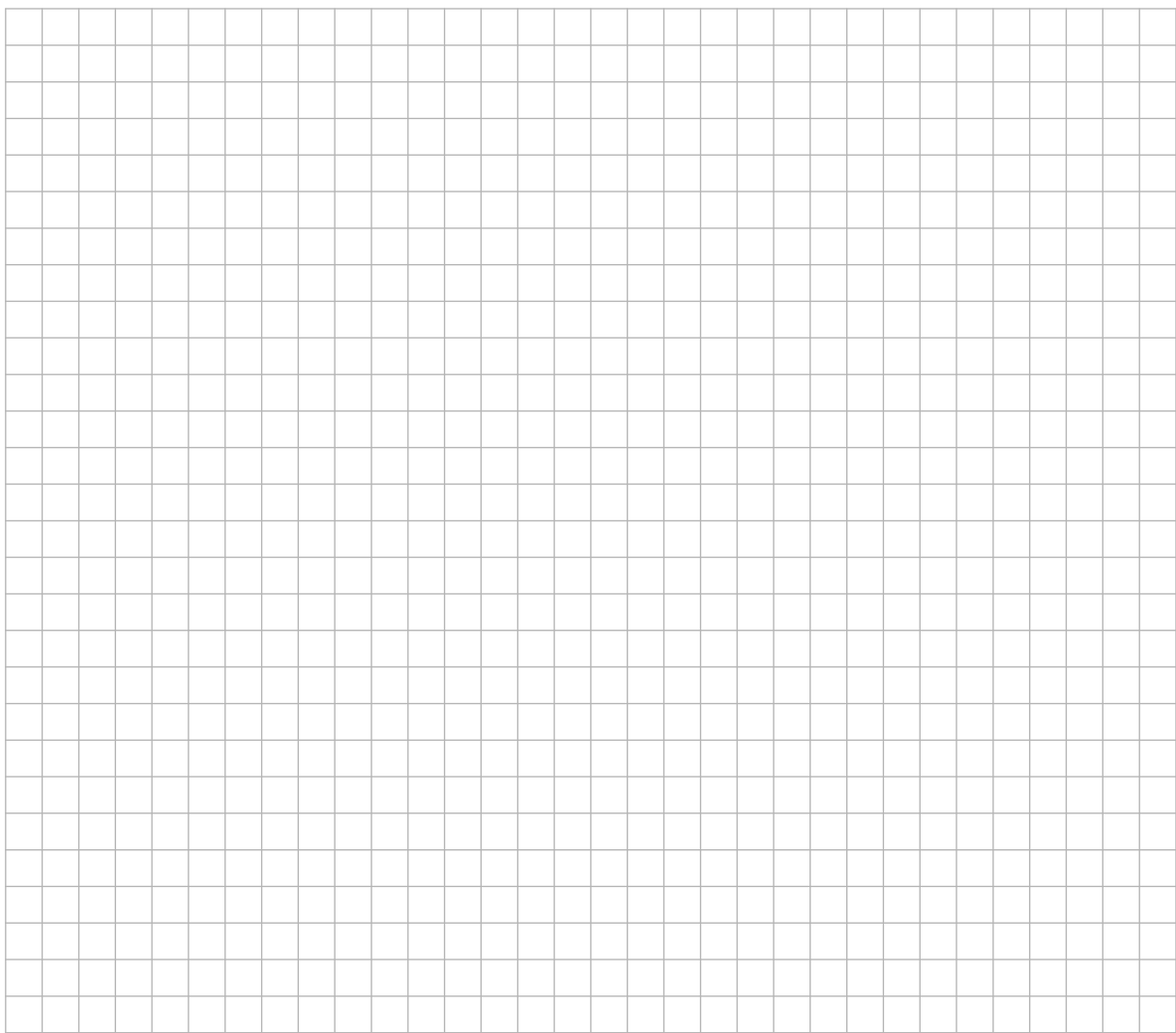
    (i) **Young's rule:** a mathematical expression used to determine a drug dosage for children. The correct dosage is calculated by dividing the child's age by an amount equal to the child's age plus 12 and then multiplying by the usual adult dose.

                                                Mosby's Dental Dictionary, 2nd edition.

Formula:

(ii) **Young's rule:** A rule for calculating the dose of medicine correct for a child by adding 12 to the child's age, dividing the sum by the child's age, then dividing the adult dose by the figure obtained.

The American Heritage Medical Dictionary

Formula:

(iii) **Young's rule:** the dose of a drug for a child is obtained by multiplying the adult dose by the child's age in years and dividing the result by the sum of the child's age plus 12.

Miller-Keane Encyclopedia and Dictionary of Medicine,
Nursing, and Allied Health, Seventh Edition.

Formula:

(c) Explain why the three formulae in **(b)** above all give the same result.

(d) The adult dose of a certain medicine is 150 mg per day. According to Young's rule, what is the correct dose for a six-year old child?

(e) Young's rule results in a certain child being given one fifth of the adult dose of a medicine. How old is this child?

page   running

(f) Another rule for working out a child's dose is based on "body surface area" (BSA). The rule is:

$$\text{child's dose} = \frac{\text{child's BSA in m}^2}{1\cdot73} \times \text{adult dose}$$

BSA is difficult to measure directly, but an estimate can be calculated from a person's height and weight. The chart below allows you to read off the BSA for a given height and weight, by drawing a straight line from the height on the left scale to the weight on the right. For example, the dotted line shows that a person of height 100 cm and weight 16 kg has a BSA of $0\cdot67$ m^2.

Height (cm)	BSA	Weight (kg)
1.80	2	80
1.70	1.9	70
1.60	1.8	60
1.50	1.7	50
1.40	1.6	40
1.30	1.5	
1.20	1.4	30
1.10	1.3	
1.00	1.2	20
90	1.1	
80	1	10
	0.9	
	0.8	
	0.7	
	0.6	
	0.5	

The correct adult dose of a certain medicine is 200 mg per day. Use the BSA rule to calculate the correct dose for a child of height 125 cm and weight 26 kg.

(g) The following apply in the case of a certain medicine and a certain child:
- the child is nine years old
- Clark's rule and Young's rule both give a dose of 90 mg per day
- the BSA rule gives a dose of 130 mg per day.

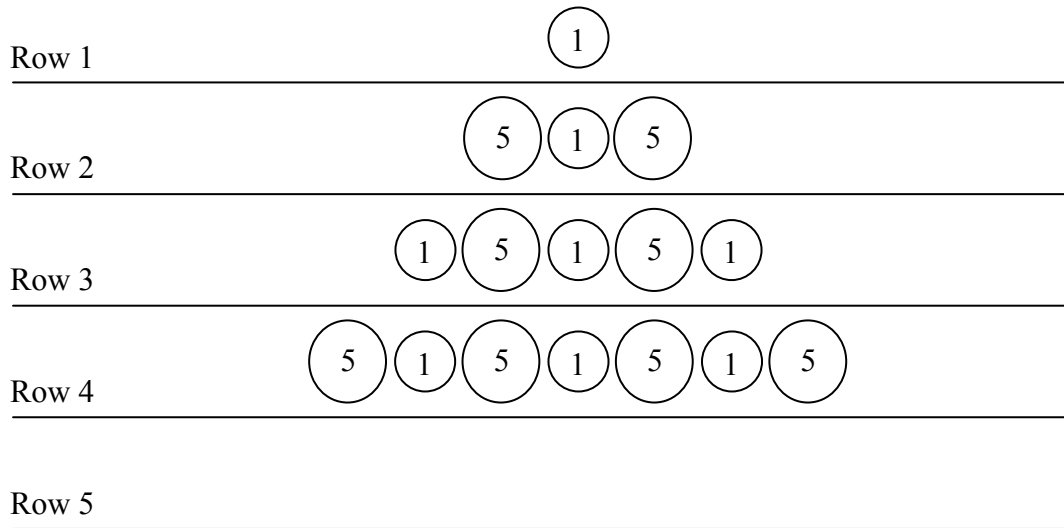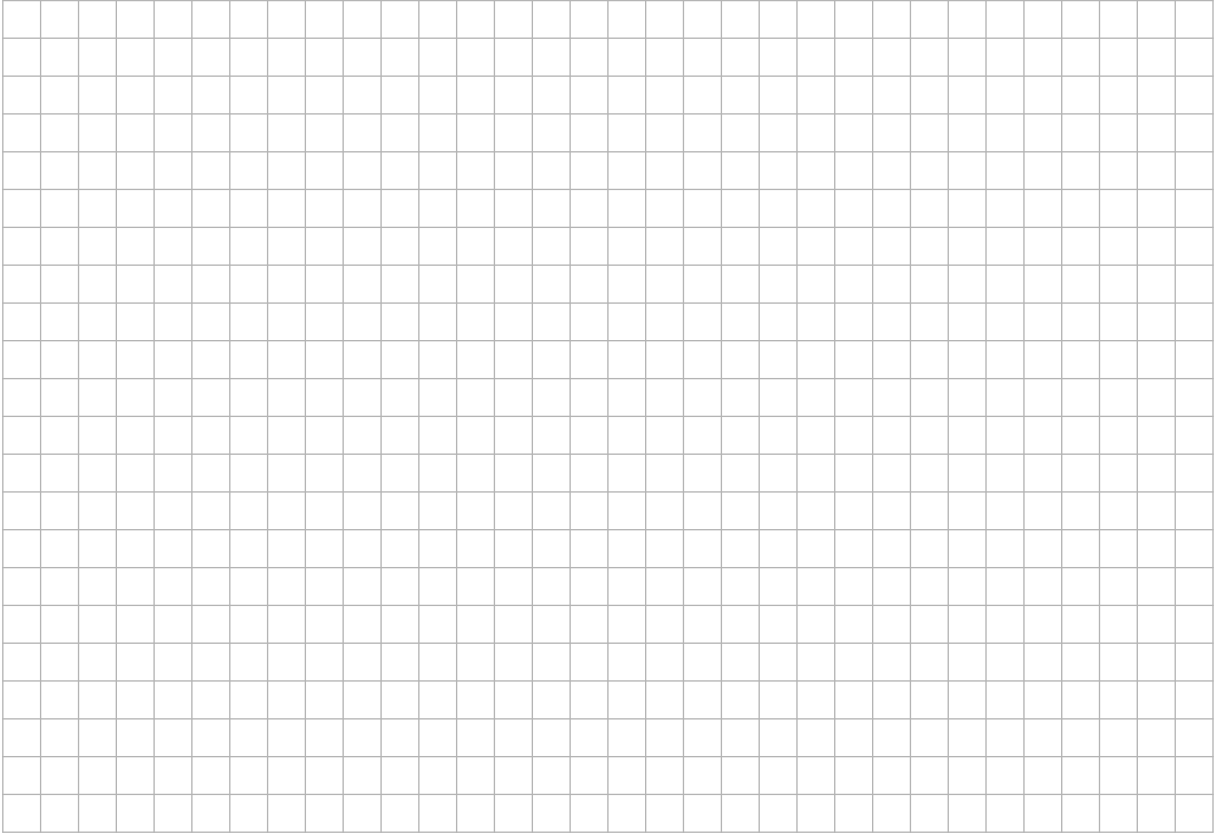Find the weight and height of this child.
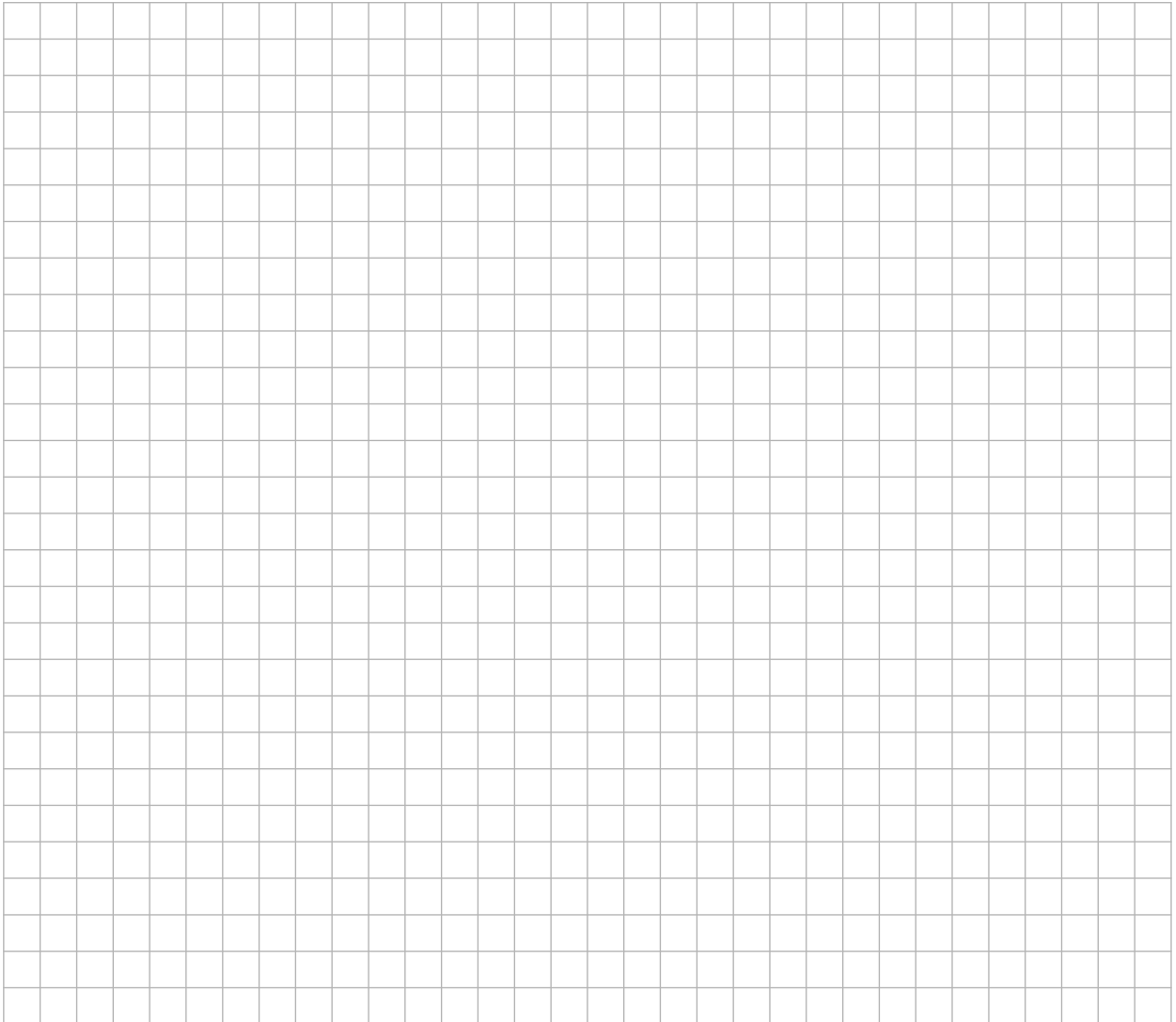
Question 8 **(50 marks)**

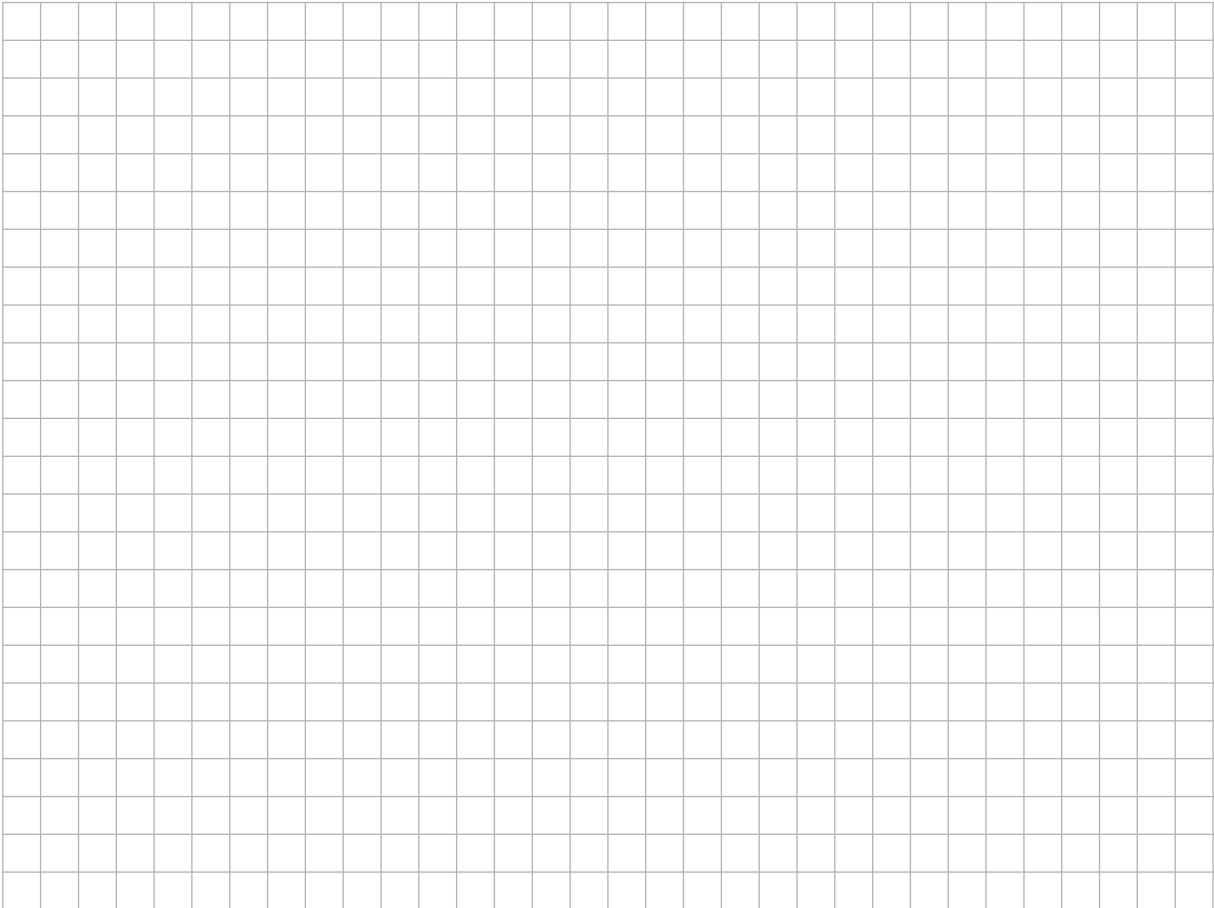Lucy is arranging 1 cent and 5 cent coins in rows. The pattern of coins in each row is as shown below.

Row 1 — ① 1

Row 2 — ⑤ ① ⑤ : 5 1 5

Row 3 — ① ⑤ ① ⑤ ① : 1 5 1 5 1

Row 4 — ⑤ ① ⑤ ① ⑤ ① ⑤ : 5 1 5 1 5 1 5

Row 5 —

(a) Draw the next row of coins above, continuing the same pattern.

(b) The table below gives the number of coins and the total value of the coins in each row. Complete the table for rows 4 to 7.

Row number n	Number of 1 cent coins	Number of 5 cent coins	Total number of coins in the row	Total value of the coins in the row
1	1	0	1	1
2	1	2	3	11
3	3	2	5	13
4				
5				
6				
7				

(c) Complete the following sentences to state, in terms of *n*, the number of 1 cent and 5 cent coins in row *n*.

(i) If *n* is odd, row *n* has _____ 1 cent coins and _____ 5 cent coins.

(ii) If *n* is even, row *n* has _____ 1 cent coins and _____ 5 cent coins.

(d) Find the total number of coins in the 40th row.

(e) Find the total value of the coins in the 40th row.

(f) Which row has coins with a total value of 337 cent?

(g) Find the total value of the coins in the first 40 rows.

Question 9 (50 marks)

(a) Investments can increase or decrease in value. The value of a particular investment of €100 was found to fit the following model:

$$V = 100 + 45t - 1{\cdot}5t^2$$

where V is the value of the investment in euro, and t is the time in months after the investment was made.

(i) Find the rate at which the value of the investment was changing after 6 months.

(ii) State whether the value of the investment was increasing or decreasing after 18 months. Justify your answer.

(iii) The investment was cashed in at the end of 24 months. How much was it worth at that time?

(iv)   How much was the investment worth when it had its maximum value?
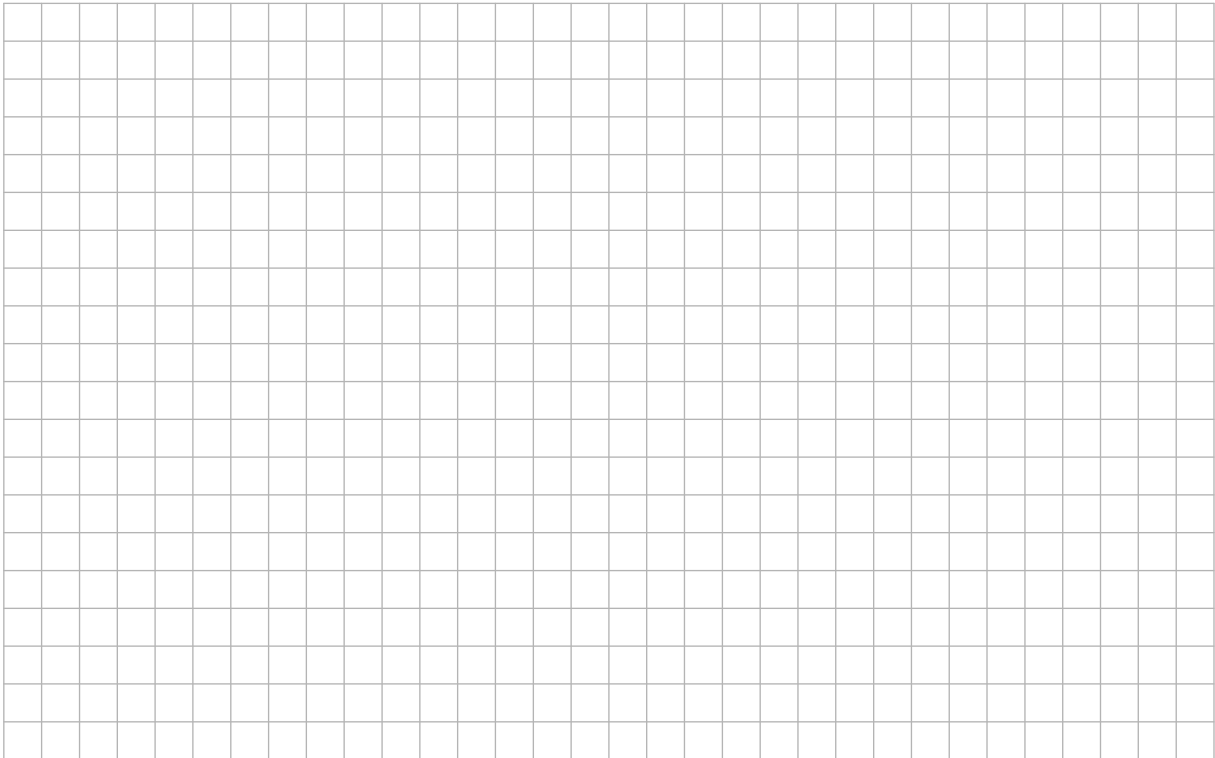
(b)   Garden paving slabs measure 40 cm by 20 cm. The slabs are to be arranged to form a rectangular paved area. There are x slabs along one side and y slabs along an adjacent side, as shown.
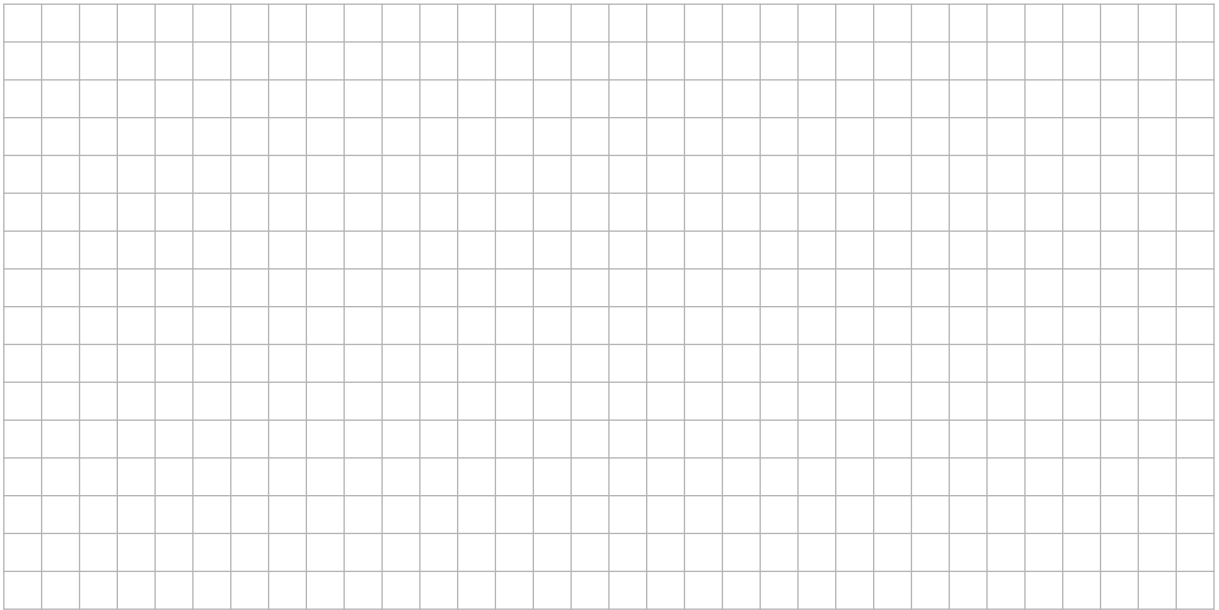
x slabs

y slabs
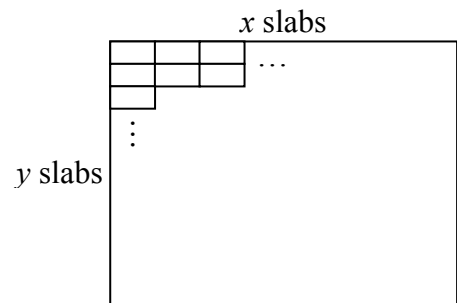
(i)   Write the length of the perimeter, in centimetres, in terms of x and y.
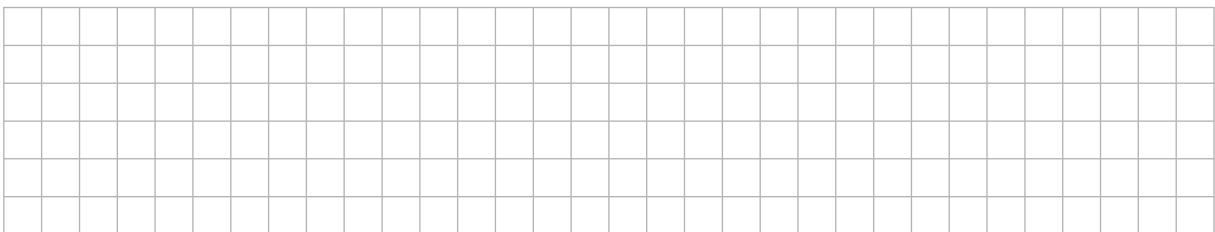
(ii)   The material being used for edging means that the perimeter is to be 64 metres. Find y in terms of x.
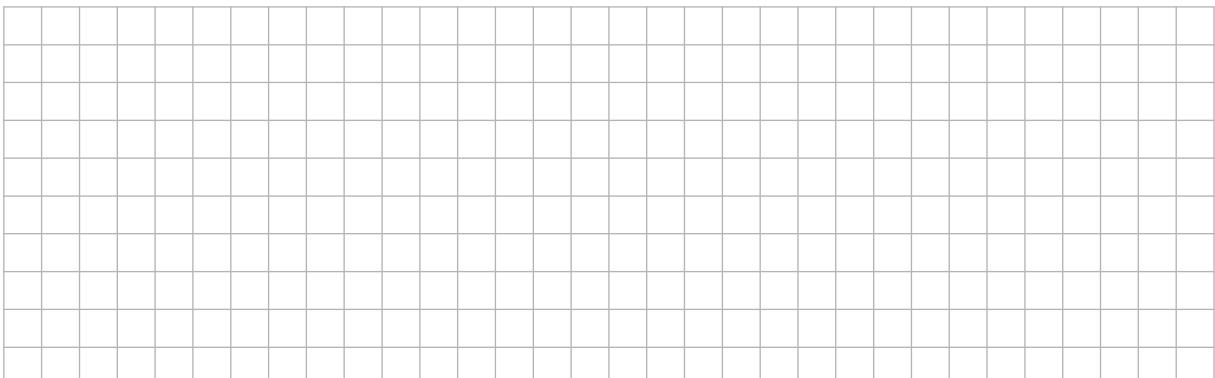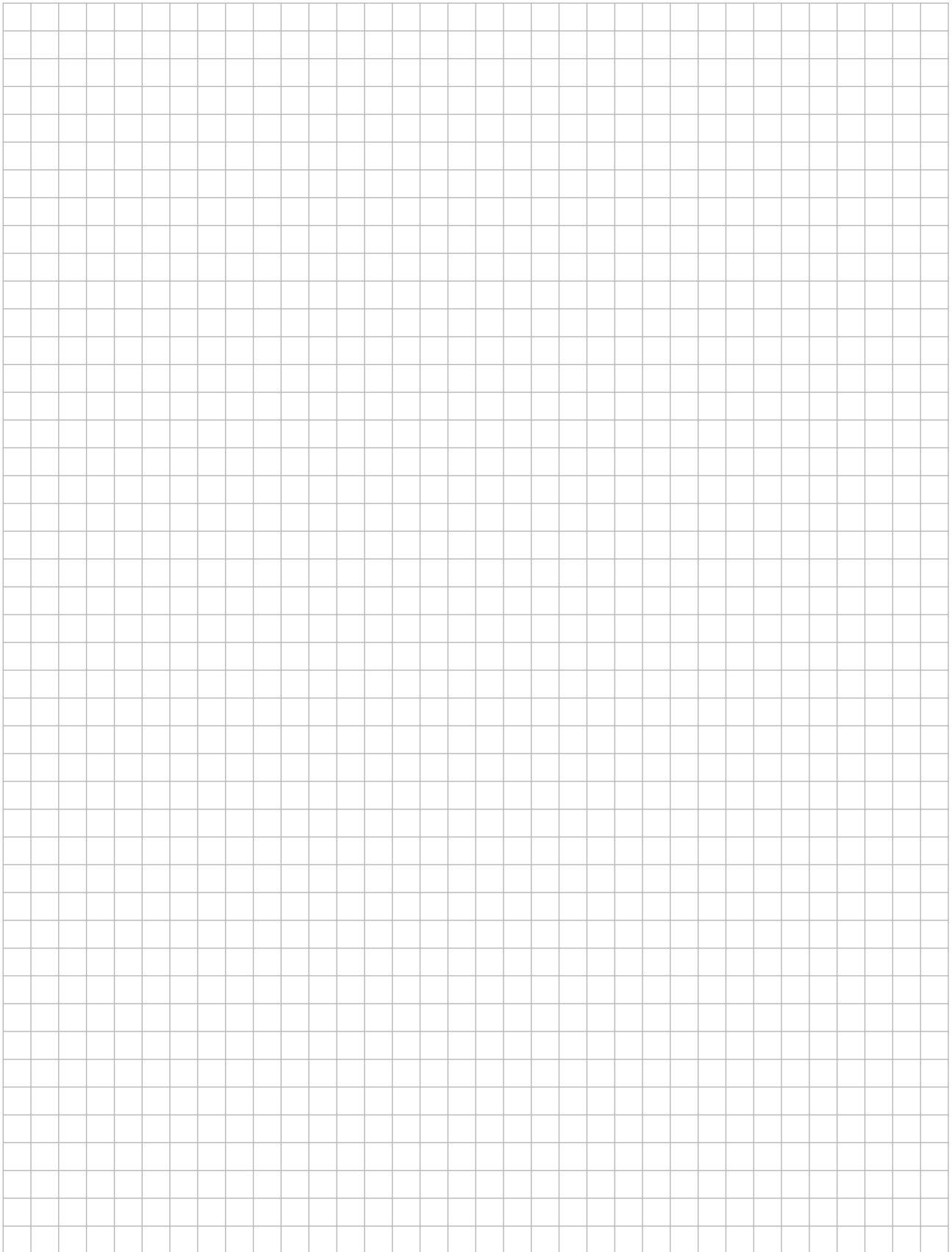
(iii) Find the value of x for which the paved area is as large as possible.

(iv) Find the number of slabs needed to pave this maximum area.
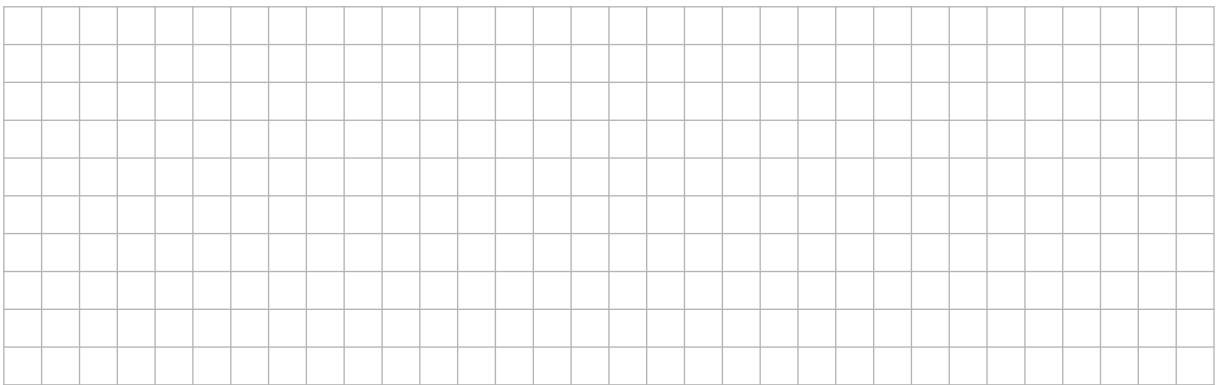
Leaving Certificate 2012 – Ordinary Level

Mathematics (Project Maths – Phase 3) – Paper 1

Friday 8 June
Afternoon 2:00 – 4:30