Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014

Mathematics
(Project Maths – Phase 3)

Paper 1

Ordinary Level

Friday 6 June Afternoon 2:00 – 4:30

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

A shopkeeper bought 25 school blazers at €30 each and 25 trousers at €20 each.

(a) Find the total cost to the shopkeeper.

(b) The shopkeeper sells a blazer and a trousers as a set for €89·95. Find her profit on this transaction.

(c) The shopkeeper sells 22 blazer and trouser sets at €89·95 each. She sells the remaining 3 sets at a discount of 20% on the selling price. Find her mark up (profit as a percentage of cost price) on the total transaction.

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Question 2 **(25 marks)**

Let $z_1 = 5 - i$ and $z_2 = 4 + 3i$, where $i^2 = -1$.

(a) **(i)** Find $z_1 - z_2$.

(ii) Verify that $|z_1 - z_2| = |z_2 - z_1|$.

(iii) Give a reason why $|z - w| = |w - z|$ will always be true, for any complex numbers z and w.

(b) Find a complex number z_3 such that $z_1 = \frac{z_2}{z_3}$.

Give your answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

Question 3 **(25 marks)**

(a) **(i)** Solve for x:

$$2(4 - 3x) + 12 = 7x - 5(2x - 7).$$

(ii) Verify your answer to **(i)** above.

(b) Solve the simultaneous equations:

$$x + y = 7$$

$$x^2 + y^2 = 25.$$

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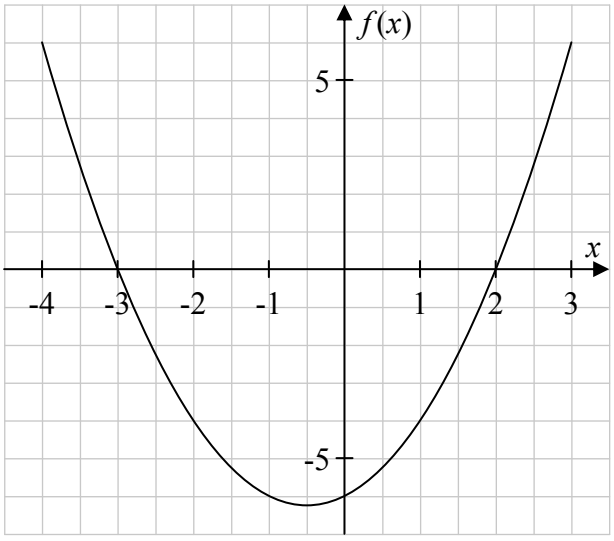
Question 4 **(25 marks)**

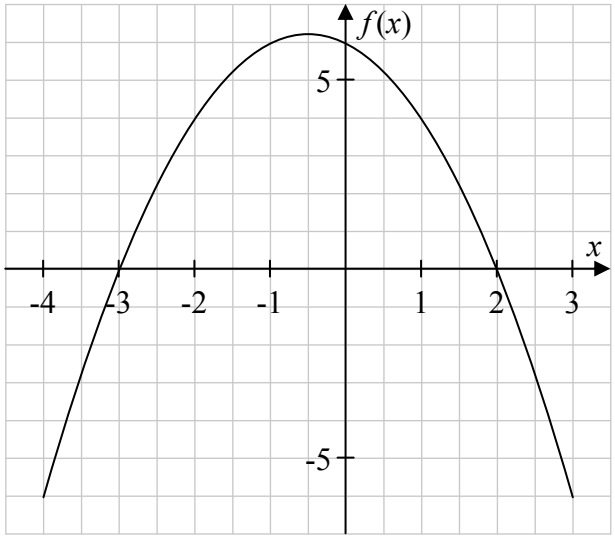
(a) Solve the equation $x^2 - x - 6 = 0$.

(b) The graphs of four quadratic functions are shown below.

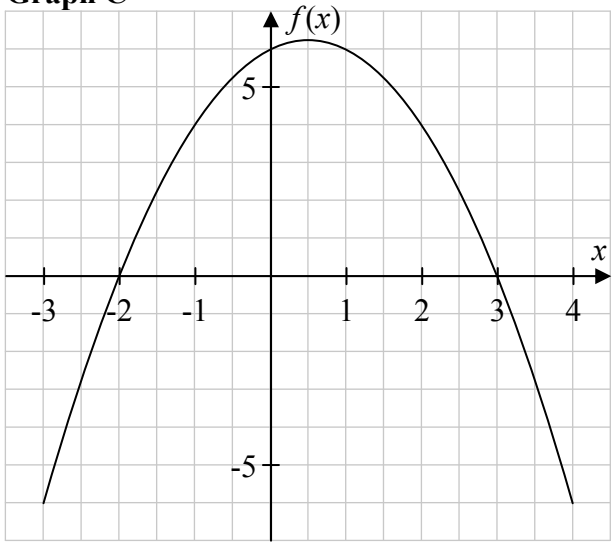
Graph A



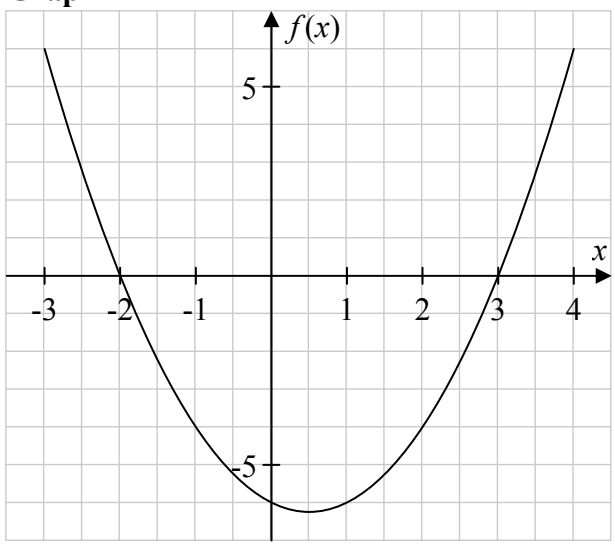
Graph B



Graph C



Graph D



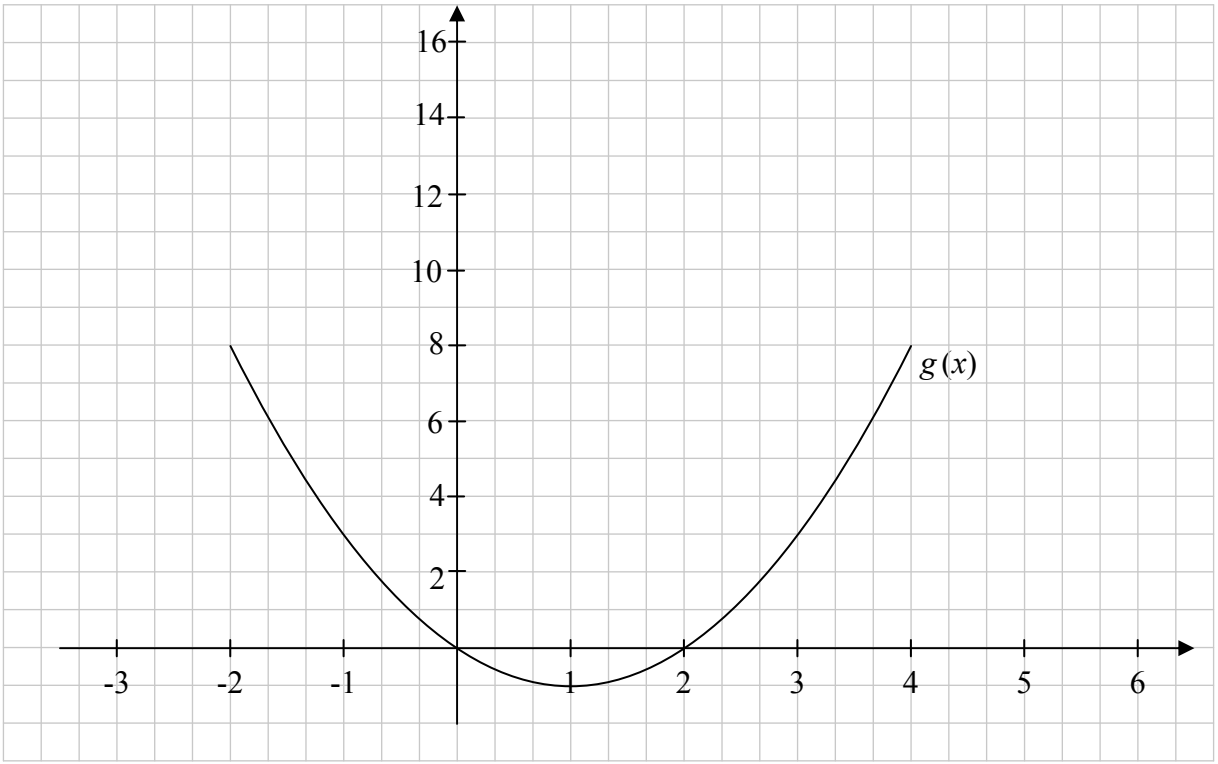
Which of the graphs above is that of the function $f : x \mapsto x^2 - x - 6$, where $x \in \mathbb{R}$?

(c) The graph of $g(x) = x^2 - 2x$, where $x \in \mathbb{R}$, is shown on the diagram below.
On the same diagram, sketch the graph of each of the functions:

(i) $h(x) = g(x) + 2$

(ii) $k(x) = g(x + 2)$.

Label each sketch clearly.



Question 5 **(25 marks)**

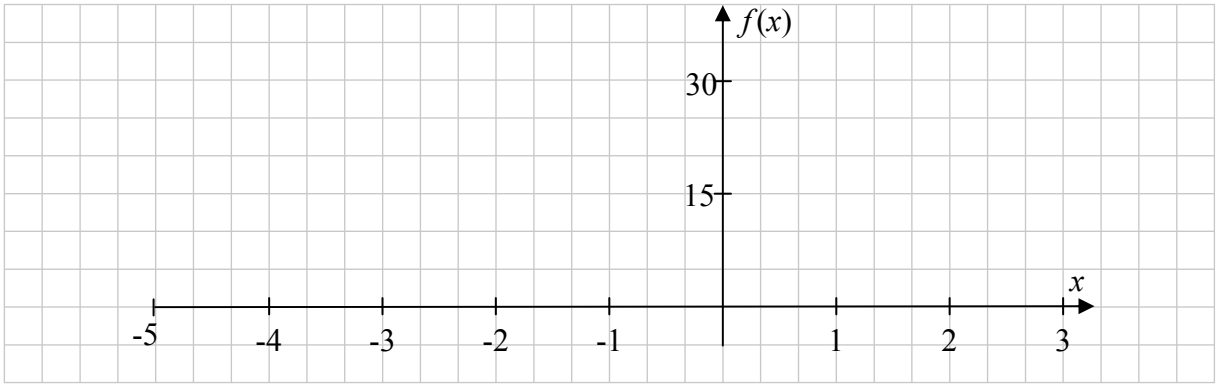
The function f is defined as $f : x \mapsto x^3 + 3x^2 - 9x + 5$, where $x \in \mathbb{R}$.

(a) **(i)** Find the co-ordinates of the point where the graph of f cuts the y-axis.

(ii) Verify that the graph of f cuts the x-axis at $x = -5$.

(b) Find the co-ordinates of the local maximum turning point and of the local minimum turning point of f.

(c) Hence, sketch the graph of the function f on the axes below.



Question 6 **(25 marks)**

The general term of an arithmetic sequence is $T_n = 15 - 2n,$ where $n \in \mathbb{N}.$

(a) **(i)** Write down the first three terms of the sequence.

(ii) Find the first negative term of the sequence.

(b) **(i)** Find $S_n = T_1 + T_2 + \cdots + T_n,$ the sum of the first *n* terms of the series, in terms of *n*.

(ii) Find the value of *n* for which the sum of the first *n* terms of the series is 0.

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Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 (35 marks)

(a) Mary bought a new car for €20 000 on the 1st July 2010.
The value of the car depreciated at a compound rate of 15% each year.
Find the value of the car, correct to the nearest euro, on the 1st July 2014.

(b) Mary wishes to buy a new car, which costs €24 000, on the 1st July 2014.

(i) *Buy Right Car Sales* offers Mary €10 500 for her old car. She can borrow the balance for one year at a rate of 11·5%. How much would she repay on 1st July 2015?

(ii) *Bargain Deals Car Sales* offers Mary €10 000 for her old car and an interest free loan of the balance for six months. At the end of the six months Mary would make a payment of €4000 and would be charged interest at a compound rate of 1·5% per month for the next six months. How much would Mary repay on 1st July 2015?

(iii) Which of the above options should Mary choose if she wishes to pay the least amount? Justify your answer by calculation.

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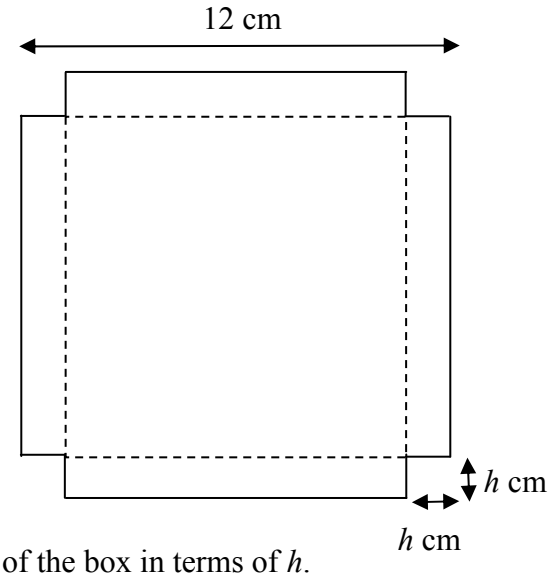
Question 8 **(40 marks)**

(a) The length of the side of a square sheet of cardboard is 12 cm. Find the area of the sheet.

(b) The diagram below shows a square sheet of cardboard of side length 12 cm, from which four small squares, each of side length h, have been removed. The sheet can be folded to form an open rectangular box of height h.



Write the length and the width of the box in terms of h.

Length of box = _____

Width of box = _____

(c) Show that the volume of the box, in terms of h, is $4h^3 - 48h^2 + 144h$.

(d) Find the value of h which gives the maximum volume of the box.

(e) Find the maximum volume of the box.

Question 9 **(75 marks)**

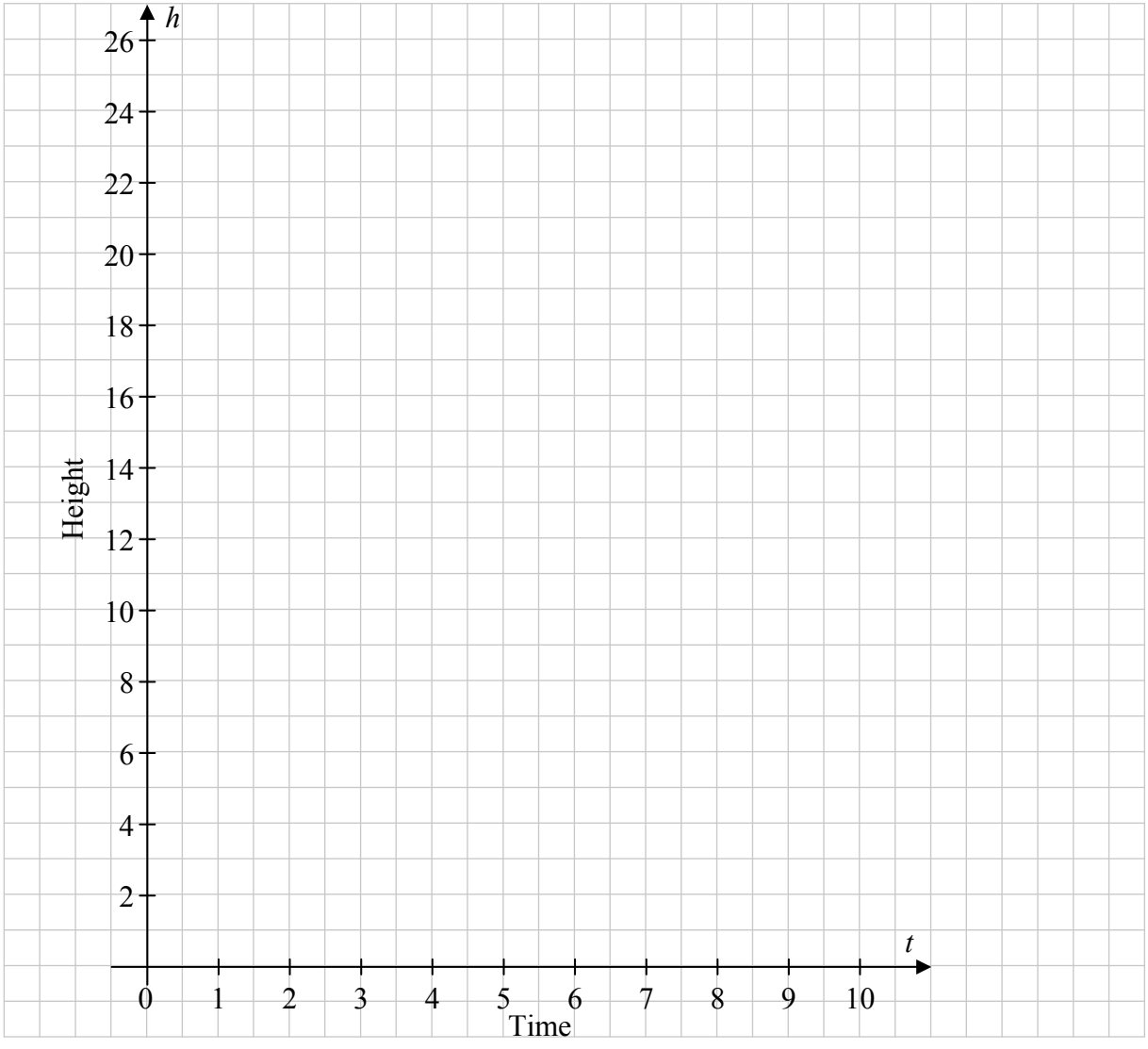
A small rocket is fired into the air from a fixed position on the ground. Its flight lasts ten seconds. The height, in metres, of the rocket above the ground after t seconds is given by

$$h = 10t - t^2.$$

(a) Complete the table below.

Time, t	0	1	2	3	4	5	6	7	8	9	10
Height, h						25	24	21	16		

(b) Draw a graph to represent the height of the rocket during the ten seconds.



(c) Use your graph to estimate:

(i) The height of the rocket after 2·5 seconds. _____

(ii) The time when the rocket will again be at this height. _____

(iii) The co-ordinates of the highest point reached by the rocket. _____

(d) **(i)** Find the slope of the line joining the points (6, 24) and (7, 21).

(ii) Would you expect the line joining the points (7, 21) and (8, 16) to be steeper than the line joining (6, 24) and (7, 21) or not? Give a reason for your answer.

(e) **(i)** Find $\frac{dh}{dt}$.

(ii) Hence, find the maximum height reached by the rocket.

(iii) Find the speed of the rocket after 3 seconds.

(f) Find the co-ordinates of the point at which the slope of the tangent to the graph is 2.

You may use this page for extra work.

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