

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2014

Marking Scheme

Mathematics
(Project Maths – Phase 3)

Ordinary Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014

Mathematics
(Project Maths – Phase 3)

Paper 1

Ordinary Level

Friday 6 June Afternoon 2:00 – 4:30

300 marks

Model Solutions – Paper 1

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1**(25 marks)**

A shopkeeper bought 25 school blazers at €30 each and 25 trousers at €20 each.

- (a) Find the total cost to the shopkeeper.

$$\begin{aligned}25 \times 30 &= 750 \\25 \times 20 &= 500 \\500 + 750 &= \text{€}1250\end{aligned}$$

- (b) The shopkeeper sells a blazer and a trousers as a set for €89.95. Find her profit on this transaction.

$$89.95 - 50 = \text{€}39.95$$

- (c) The shopkeeper sells 22 blazer and trouser sets at €89.95 each. She sells the remaining 3 sets at a discount of 20% on the selling price. Find her mark up (profit as a percentage of cost price) on the total transaction.

$$\begin{aligned}22 \times 89.95 &= \text{€}1978.90 \\89.95 \times 0.8 &= \text{€}71.96 \\3 \times 71.96 &= \text{€}215.88 \\1978.90 + 215.88 &= \text{€}2194.78 \\2194.78 - 1250 &= 944.78 \text{ (Profit)} \\ \frac{944.78}{1250} \times 100 &= 75.58\%\end{aligned}$$

Question 2**(25 marks)**Let $z_1 = 5 - i$ and $z_2 = 4 + 3i$, where $i^2 = -1$.**(a) (i)** Find $z_1 - z_2$.

$$\begin{aligned}
 &5 - i - (4 + 3i) \\
 &5 - i - 4 - 3i \\
 &1 - 4i
 \end{aligned}$$

(ii) Verify that $|z_1 - z_2| = |z_2 - z_1|$.

$$\begin{aligned}
 |1 - 4i| &= \sqrt{1^2 + (-4)^2} = \sqrt{17} \\
 z_2 - z_1 &= 4 + 3i - (5 - i) \\
 &= 4 + 3i - 5 + i \\
 &= -1 + 4i \\
 |-1 + 4i| &= \sqrt{(-1)^2 + 4^2} = \sqrt{17}
 \end{aligned}$$

(iii) Give a reason why $|z - w| = |w - z|$ will always be true, for any complex numbers z and w .

The absolute values of the real and imaginary parts are the same. Therefore $\sqrt{a^2 + b^2}$ will be the same.

or

One complex number is the image of the other in S_o . Therefore they must both be the same distance from the origin.

(b) Find a complex number z_3 such that $z_1 = \frac{z_2}{z_3}$.Give your answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

$$\begin{aligned}
 z_3 &= \frac{z_2}{z_1} & z_3 z_1 &= z_2 \\
 & & \text{Let } z_3 &= a + bi \\
 & & \Rightarrow (a + bi)(5 - i) &= 4 + 3i \\
 & & \Rightarrow 5a + b &= 4 \\
 & & 5b - a &= 3 \\
 & & \Rightarrow a = \frac{17}{26} \text{ and } b = \frac{19}{26}
 \end{aligned}$$

or

$$\begin{aligned}
 &\frac{(4 + 3i)(5 + i)}{(5 - i)(5 + i)} \\
 &= \frac{20 + 19i + 3i^2}{25 - i^2} \\
 &= \frac{17 + 19i}{26} = \frac{17}{26} + \frac{19}{26}i
 \end{aligned}$$

Question 3**(25 marks)****(a) (i)** Solve for x :

$$2(4 - 3x) + 12 = 7x - 5(2x - 7).$$

$$8 - 6x + 12 = 7x - 10x + 35$$

$$-15 = 3x$$

$$x = -5$$

(ii) Verify your answer to **(i)** above.

$$x = -5$$

$$2(4 - (-15)) + 12$$

$$38 + 12$$

$$50$$

$$7(-5) - 5(-10 - 7)$$

$$-35 + 85$$

$$50$$

$$[50 = 50]$$

(b) Solve the simultaneous equations:

$$x + y = 7$$

$$x^2 + y^2 = 25.$$

$$x = 7 - y$$

$$(7 - y)^2 + y^2 = 25$$

$$y^2 - 7y + 12 = 0$$

$$(y - 4)(y - 3) = 0$$

$$y = 4 \quad y = 3$$

$$x = 7 - 4 \quad x = 7 - 3$$

$$x = 3 \quad x = 4$$

$$(3, 4) \quad (4, 3)$$

Question 4

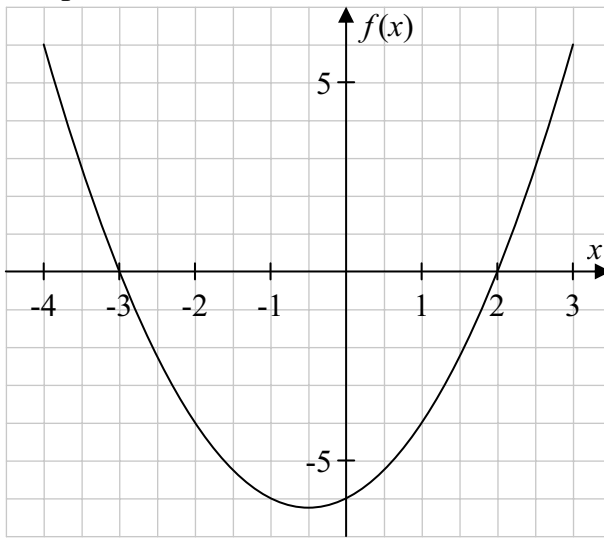
(25 marks)

(a) Solve the equation $x^2 - x - 6 = 0$.

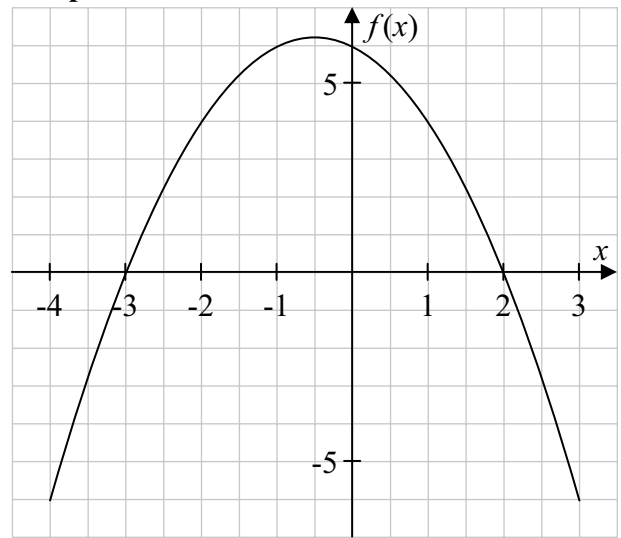
$(x+2)(x-3) = 0$ $x = -2 \quad x = 3$	or	$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2}$ $x = \frac{1 \pm \sqrt{25}}{2}$ $x = 3 \quad x = -2$
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(b) The graphs of four quadratic functions are shown below.

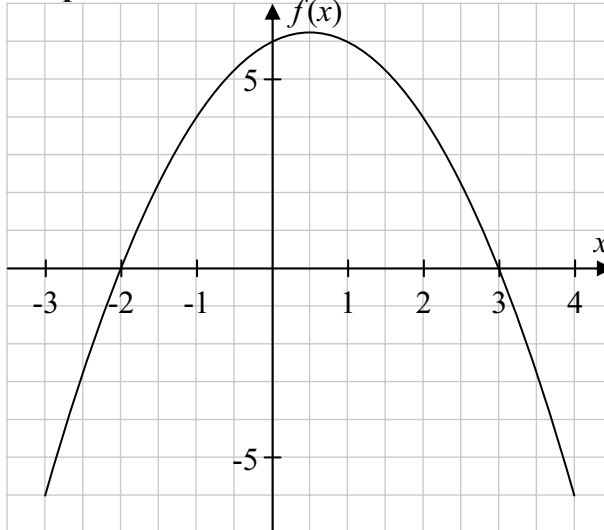
Graph A



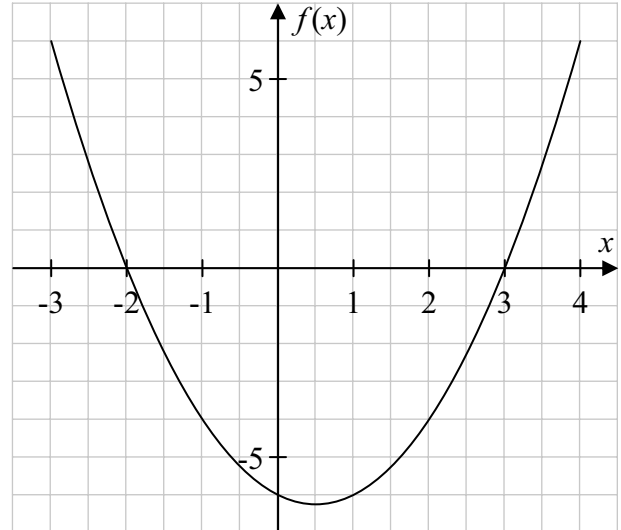
Graph B



Graph C



Graph D



Which of the graphs above is that of the function $f : x \mapsto x^2 - x - 6$, where $x \in \mathbb{R} ? \mathbb{N}$

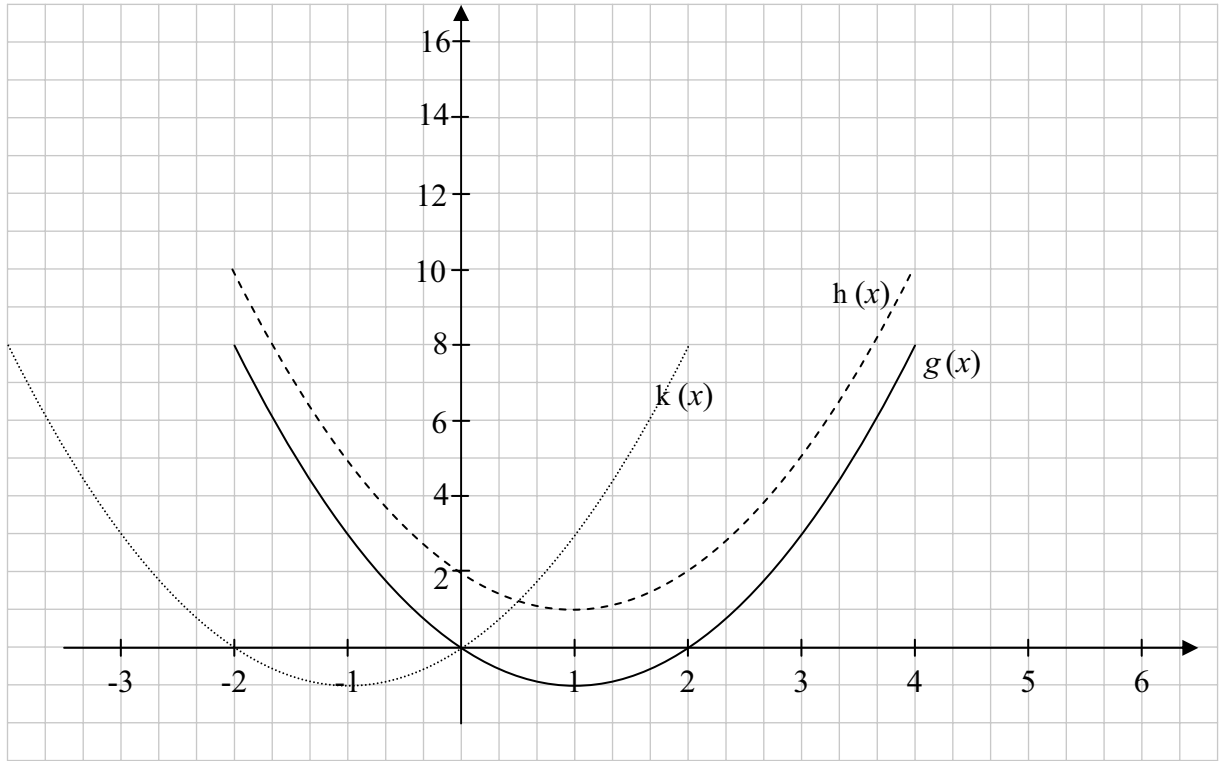
Graph D

(c) The graph of $g(x) = x^2 - 2x$, where $x \in \mathbb{R}$, is shown on the diagram below. On the same diagram, sketch the graph of each of the functions:

(i) $h(x) = g(x) + 2$

(ii) $k(x) = g(x+2)$.

Label each sketch clearly.



Question 5**(25 marks)**

The function f is defined as $f : x \mapsto x^3 + 3x^2 - 9x + 5$, where $x \in \mathbb{R}$.

- (a) (i) Find the co-ordinates of the point where the graph of f cuts the y -axis.

$$\begin{aligned} f(0) &= 0^3 + 3(0)^2 - 9(0) + 5 \\ f(0) &= 5 \\ &[(0, 5)] \end{aligned}$$

- (ii) Verify that the graph of f cuts the x -axis at $x = -5$.

$$\begin{aligned} f(-5) &= (-5)^3 + 3(-5)^2 - 9(-5) + 5 \\ f(-5) &= 0 \end{aligned}$$

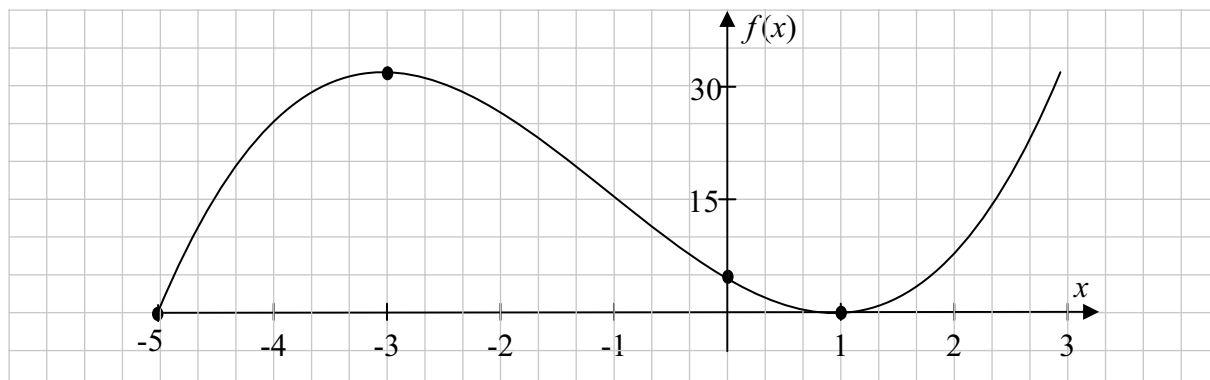
- (b) Find the co-ordinates of the local maximum turning point and of the local minimum turning point of f .

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ 3x^2 + 6x - 9 &= 0 \\ x^2 + 2x - 3 &= 0 \\ (x+3)(x-1) &= 0 \\ x &= -3 \text{ or } x = 1 \end{aligned}$$

$$\begin{aligned} f(-3) &= (-3)^3 + 3(-3)^2 - 9(-3) + 5 \\ y &= 32 \\ &(-3, 32) \\ f(1) &= (1)^3 + 3(1)^2 - 9(1) + 5 \\ y &= 0 \\ &(1, 0) \end{aligned}$$

Local Max $(-3, 32)$ Local Min $(1, 0)$

- (c) Hence, sketch the graph of the function f on the axes below.



Question 6**(25 marks)**

The general term of an arithmetic sequence is $T_n = 15 - 2n$, where $n \in \mathbb{N}$.

- (a) (i) Write down the first three terms of the sequence.

$$\begin{array}{lll} T_1 = 15 - 2(1) & T_2 = 15 - 2(2) & T_3 = 15 - 2(3) \\ T_1 = 13 & T_2 = 11 & T_3 = 9 \end{array}$$

Note: Accept T_0, T_1, T_2 15, 13, 11

- (ii) Find the first negative term of the sequence.

$$\begin{array}{ll} 15 - 2n < 0 & \\ -2n < -15 & \\ 2n > 15 & \text{or} \quad -1 \\ n > 7\frac{1}{2} & \\ \Rightarrow \text{The } 8^{\text{th}} \text{ term} & \end{array}$$

- (b) (i) Find $S_n = T_1 + T_2 + \dots + T_n$, the sum of the first n terms of the series, in terms of n .

$$S_n = \frac{n}{2}(2(13) + (n-1)(-2))$$

- (ii) Find the value of n for which the sum of the first n terms of the series is 0.

$$\begin{array}{l} S_n = \frac{n}{2}(-2n + 28) = [-n^2 + 14n] \\ \frac{n}{2}(-2n + 28) = 0 \\ n(-2n + 28) = 0 \\ -2n + 28 = 0 \\ n = 14 \end{array}$$

Answer **all three** questions from this section.

Question 7**(35 marks)**

- (a) Mary bought a new car for €20 000 on the 1st July 2010.
The value of the car depreciated at a compound rate of 15% each year.
Find the value of the car, correct to the nearest euro, on the 1st July 2014.

$$20\,000(1 - 0.15)^4 = \text{€}10\,440$$

- (b) Mary wishes to buy a new car, which costs €24 000, on the 1st July 2014.
- (i) *Buy Right Car Sales* offers Mary €10 500 for her old car. She can borrow the balance for one year at a rate of 11.5%. How much would she repay on 1st July 2015?

$$24\,000 - 10\,500 = \text{€}13\,500 \text{ to borrow}$$

$$13\,500 \times 1.115 = \text{€}15\,052.50 \text{ to repay on 1}^{\text{st}} \text{ July 2015}$$

Note: $13\,500 \times 11.5\% = 1552.50$
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- (ii) *Bargain Deals Car Sales* offers Mary €10 000 for her old car and an interest free loan of the balance for six months. At the end of the six months Mary would make a payment of €4000 and would be charged interest at a compound rate of 1.5% per month for the next six months. How much would Mary repay on 1st July 2015?

$$24\,000 - 10\,000 = €14\,000 \text{ Loan}$$

$$14\,000 - 4000 = 10\,000 \text{ To repay after 6 months}$$

$$10\,000 \times (1.015)^6 = €10\,934.43 \text{ to repay on 1}^{\text{st}} \text{ July 2015}$$

- (iii) Which of the above options should Mary choose if she wishes to pay the least amount? Justify your answer by calculation.

$$\text{Total repayment } \textit{Buy Right} \quad €15\,052.50$$

$$\text{Total repayment } \textit{Bargain deals} \quad 10\,934.43 + 4000 = €14\,934.43$$

$$[\text{Difference } €118.07]$$

She would pay less overall if she bought the car from *Bargain Deals* **(option (ii))**

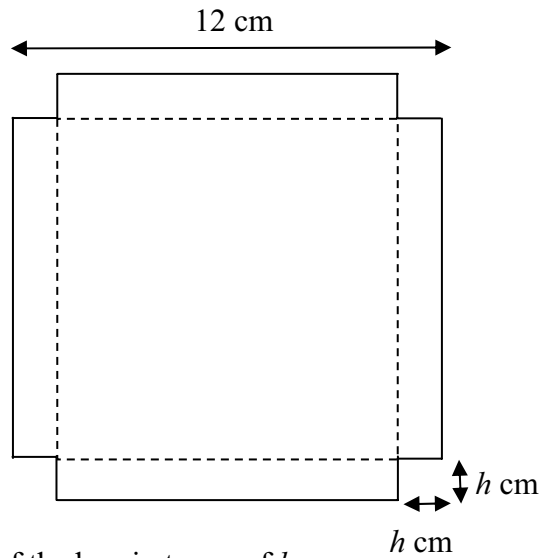
Question 8

(40 marks)

- (a) The length of the side of a square sheet of cardboard is 12 cm. Find the area of the sheet.

$$12 \times 12 = 144 \text{ cm}^2$$

- (b) The diagram below shows a square sheet of cardboard of side length 12 cm, from which four small squares, each of side length h , have been removed. The sheet can be folded to form an open rectangular box of height h .



Write the length and the width of the box in terms of h .

Length of box =

$$12 - 2h$$

Width of box =

$$12 - 2h$$

- (c) Show that the volume of the box, in terms of h , is $4h^3 - 48h^2 + 144h$.

$$\begin{aligned} V &= (12 - 2h)(12 - 2h)h \\ V &= (144 - 48h + 4h^2)h \\ V &= 4h^3 - 48h^2 + 144h \end{aligned}$$

- (d) Find the value of h which gives the maximum volume of the box.

$$\frac{dV}{dh} = 12h^2 - 96h + 144$$

$$12h^2 - 96h + 144 = 0$$

$$h^2 - 8h + 12 = 0$$

$$(h-2)(h-6) = 0$$

$$h = 2 \quad [h = 6]$$

- (e) Find the maximum volume of the box.

$$V = 8 \times 8 \times 2$$

$$V = 128 \text{ cm}^3$$

Question 9**(75 marks)**

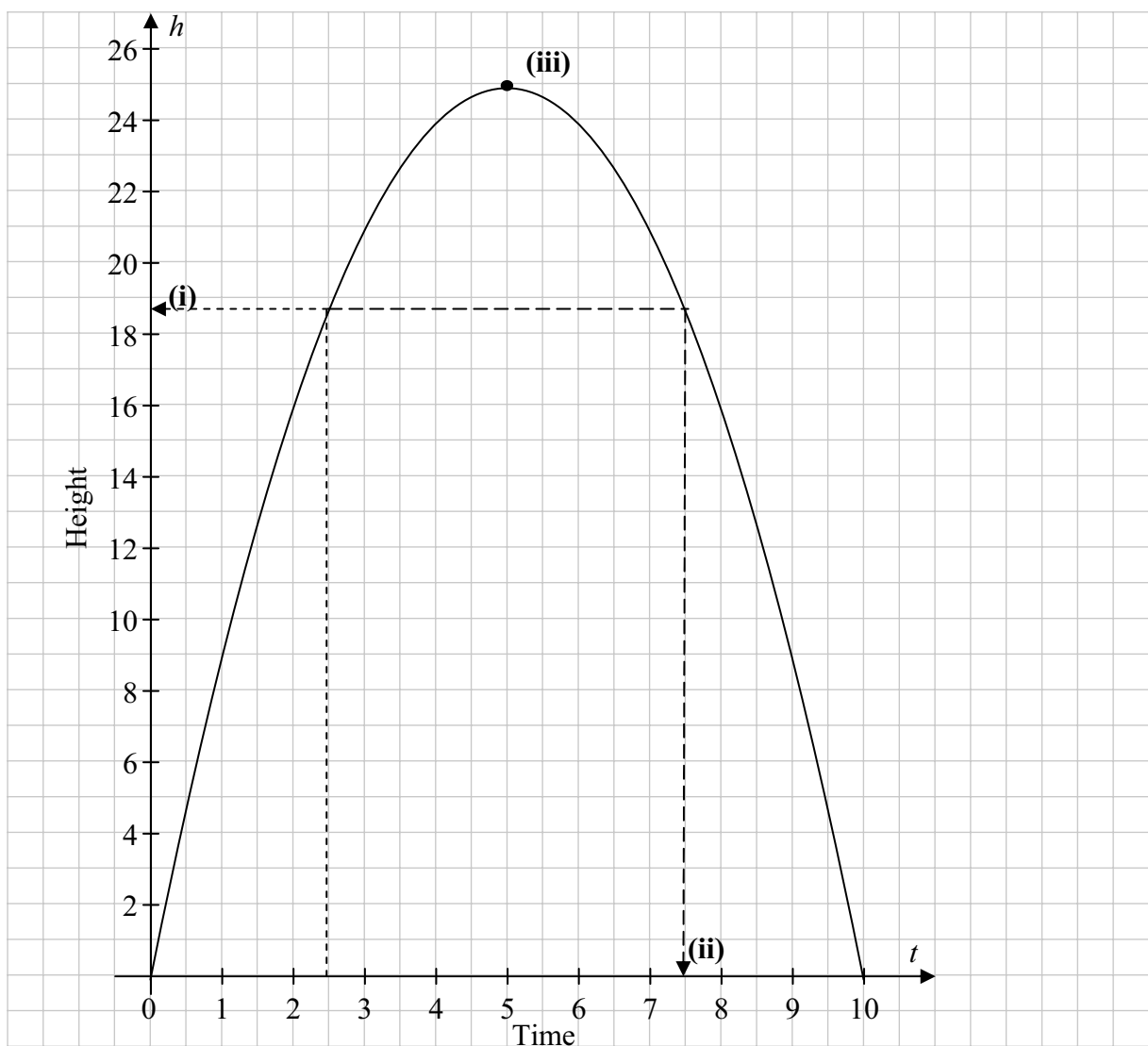
A small rocket is fired into the air from a fixed position on the ground. Its flight lasts ten seconds. The height, in metres, of the rocket above the ground after t seconds is given by

$$h = 10t - t^2.$$

(a) Complete the table below.

Time, t	0	1	2	3	4	5	6	7	8	9	10
Height, h	0	9	16	21	24	25	24	21	16	9	0

(b) Draw a graph to represent the height of the rocket during the ten seconds.



(c) Use your graph to estimate:

(i) The height of the rocket after 2.5 seconds.

19 m

(ii) The time when the rocket will again be at this height.

7.5 s

(iii) The co-ordinates of the highest point reached by the rocket.

(5, 25)

(d) (i) Find the slope of the line joining the points (6, 24) and (7, 21).

$$m = \frac{21 - 24}{7 - 6} = -3$$

(ii) Would you expect the line joining the points (7, 21) and (8, 16) to be steeper than the line joining (6, 24) and (7, 21) or not? Give a reason for your answer.

$$m = \frac{16 - 21}{8 - 7} = -5$$

Yes. The line is steeper because the absolute value of the slope is greater.

- (e) (i) Find $\frac{dh}{dt}$.

$$\frac{dh}{dt} = 10 - 2t$$

- (ii) Hence, find the maximum height reached by the rocket.

$$\begin{aligned}10 - 2t &= 0 \\ t &= 5 \\ h &= 10(5) - 5^2 = 25 \text{ m}\end{aligned}$$

- (iii) Find the speed of the rocket after 3 seconds.

$$\begin{aligned}S &= \frac{dh}{dt} = 10 - 2(3) \\ \frac{dh}{dt} &= 4 \text{ m/s}\end{aligned}$$

- (f) Find the co-ordinates of the point at which the slope of the tangent to the graph is 2.

$$\begin{aligned}10 - 2t &= 2 \\ t &= 4 \\ h &= 10(4) - 4^2 = 24 \\ &[(4, 24)]\end{aligned}$$

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 3, 5		
10 mark scales	0, 10	0, 5, 10	0, 3, 7, 10	0, 3, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded.

Summary of mark allocations and scales to be applied

Section A

Question 1	
(a)	10C
(b)	5B
(c)	10D
Question 2	
(a) (i)	10C
(a) (ii)	5C
(a) (iii)	5B
(b)	5C
Question 3	
(a) (i)	10C
(a)(ii)	5B
(b)	10D
Question 4	
(a)	15C
(b)	5A
(c)	10C
Question 5	
(a) (i)(ii)	10C
(b)	10D
(c)	5C
Question 6	
(a) (i)	10C
(a)(ii)	5C
(b) (i)	5C
(b) (ii)	5C

Section B

Question 7	
(a)	10C
(b) (i)	10C
(b) (ii)	5C
(b) (iii)	10B
Question 8	
(a)	5B
(b)	10C
(c)	10C
(d)	10D
(e)	5B
Question 9	
(a)	15C
(b)	15C
(c) (i)	5B
(c) (ii)	5B
(c) (iii)	5B
(d) (i)(ii)	10D
(e) (i)	5B
(e) (ii)	5C
(e) (iii)	5B
(f)	5C

Detailed marking notes

NOTE: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.

Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.

Section A

Question 1

(a) Scale 10C

Low partial credit:

- Finds cost of blazers **or** trousers
- Finds cost of 1 blazer and trouser set

High partial credit:

- Finds cost of 25 of each

(b) Scale 5B

Partial credit:

- Subtracts either 20 or 30 from 89.95
- Cost of 1 set = €50

(c) Scale 10D

Low partial credit:

- Finds SP of 22 sets
- Finds 20% of some figure

Middle partial credit:

- Finds total selling price
- Finds total profit

High partial credit:

- Correct expression for mark-up written but fails to finish or finishes incorrectly
- Incorrect total profit, from substantially correct work, but finishes correctly

Question 2

(a)(i) Scale 10C

Low partial credit:

- Correct substitution for z_1 or z_2 in expression

High partial credit:

- Correct substitution for z_1 and z_2 in expression but error in finishing

(a)(ii) Scale 5C

Low partial credit:

- Use of candidate's answer from (i)
- Correct modulus formula written
- $z_2 - z_1$ substituted

High partial credit:

- One modulus correctly calculated

(a)(iii) Scale 5B

Partial credit:

- Incomplete explanation but mentions distance or length or squaring
- $z - w = -(w - z)$ in words or similar

(b) Scale 5C

Low partial credit:

- Terms rearranged
- Correct substitution
- $z_3 = z_1 z_2$ and continues

High partial credit:

- Multiplication above and below by correct conjugate and some subsequent work
- Correct application of Real = Real, Im = Im
- $z_3 = \frac{z_1}{z_2}$ and continues

Question 3

(a)(i) Scale 10C

Low partial credit:

- Some correct attempt at solving

High partial credit:

- Not more than 2 errors in solving

(a)(ii) Scale 5B

Partial credit:

- Some correct substitution of answers from (i)
- No conclusion or incorrect conclusion if not equal

(b) Scale 10D

Low partial credit:

- Some correct attempt at isolating x or y in linear equation
- Trial and Error into both equations (once only)

Middle partial credit:

- Correctly substitutes into quadratic
- Trial and Error into both equations (more than once)

High partial credit:

- Solves correctly for 1 variable only and stops or one (x, y) only
- Trial and Error into both equations leading to correct (x, y)

Question 4

(a) Scale 15 C

Low partial credit:

- Some correct effort at factorising
- Correct quadratic formula written
- Trial and Error with no correct solution

High partial credit:

- Correct factors found
- Solutions found from incorrect, but plausible, factors
- Correct substitution into quadratic formula
- Solutions found from substantially correct use of formula
- Trial and Error leading to one correct solution (verified)

(b) Scale 5A

Linked to (a)

(c) Scale 5C

Low partial credit:

- 1 or 2 graphs of correct shape drawn
- Effort at evaluating $h(x)$ and/or $k(x)$

High partial credit:

- Both drawn correctly but not labelled or incorrectly labelled
- $h(x) = g(x) - 2$ and/or $k(x) = g(x - 2)$ drawn and labelled
- 1 graph drawn correctly and labelled

Question 5

(a)(i)(ii) Scale 10C

Low partial credit:

- Some correct substitution in either or both expressions

High partial credit:

- One expression worked out correctly
- Both expressions worked out but with errors

(b) Scale 10D

Low partial credit:

- Differentiates at least one term correctly
- States $f'(x) = 0$

Middle partial credit:

- Correct differentiation and stops
- Correct structure to $f'(x) = 0$ but with errors, and continues
- Correct max and min found graphically

High partial credit:

- Finds x values only of turning points
- Finds co-ordinates but with 2 or more errors

(c) Scale 5C

Low partial credit:

- Plots 2 or 3 relevant points correctly
- Sketches the correct shape with no reference to relevant points

High partial credit:

- Plots 4 relevant points correctly, but does not sketch a curve, or sketches an incorrectly shaped curve

Question 6

(a)(i) Scale 10C

Low partial credit:

- Finds one term only
- Some correct substitution

High partial credit:

- Finds 2 terms correctly
- Correct substitution for all 3 terms but not evaluated

(a)(ii) Scale 5C

Low partial credit:

- Insufficient listing, but must list beyond T_3
- Some attempt at substitution
- Lets $T_n = 0$ or $T_n > 0$

High partial credit:

- Stops at $n > 7\frac{1}{2}$ or $n = 7\frac{1}{2}$
- List method, but fails to identify term or number of the terms
- Sign error when solving inequality

(b)(i) Scale 5C

Low partial credit:

- Correct formula written
- Identifies a and/or d

High partial credit:

- Correct formula substituted with 1 error

(b)(ii) Scale 5C

Low partial credit:

- Some correct simplification of expression for S_n carried to (b)(ii)
- Solves $T_n = 0$ correctly

High partial credit:

- Writes **correct** (quadratic) expression for $S_n = 0$

Section B

Question 7

(a) Scale 10C

Low partial credit:

- Writes correct formula without further work of merit
- Use of appreciation method
- Calculates 15% of an amount correctly

High partial credit:

- Correct substitution into correct formula but errors in working out
- Calculates the amount at the end of each of 4 years, but with errors
- Calculates the amount at the end of each of 3 years, and stops

(b)(i) Scale 10C

Low partial credit:

- Calculates the balance to be borrowed and stops
- Calculates 11.5% of an amount correctly

High partial credit:

- Calculates interest only
- Calculates amount before deducting €10 500

(b)(ii) Scale 5C

Low partial credit:

- Calculates amount to be repaid at end of 6 months
- Calculates 1.5% of an amount correctly
- Uses Simple Interest

High partial credit:

- Correct substitution into correct formula but with errors
- Calculates 1.5% of an incorrect relevant figure correctly for 6 months
- Principal correct and calculates correctly for 3 or more months

(b)(iii) Scale 10B

Partial credit:

- Correct conclusion with no justification
- Correct conclusion from incorrect total repayment(s)

Question 8

(a) Scale 5B

Partial credit:

- 12×12 and stops

(b) Scale 10C

Low partial credit:

- $2h$ written
- $12 -$ incorrect h
- $12 + 2h$

High partial credit:

- 1 value correct
- Width = length (where length was of some merit e.g. $12 - h$ in both)

(c) Scale 10C

Low partial credit:

- Correct formula written
- Some relevant attempt at multiplying terms

High partial credit:

- $l \times w \times h$ fully substituted and stops
- Errors in finding product

Note: If l and w in (b) were without merit LPC at most

(d) Scale 10D

Low partial credit:

- One or two terms differentiated correctly
- Value found by trial and error

Middle partial credit:

- Correct differentiation and stops
- Correct structure to $f'(x) = 0$ but with errors, and continues
- Correct max found graphically

High partial credit:

- Correct derivative = 0 and stops or error(s) in solving it

(e) Scale 5B

Partial credit:

- Value of h from (d) substituted into $V(h)$
- More than one effort at trial and error
- $l \times w \times h$ with some correct or relevant substitution

Question 9

(a) Scale 15C

Low partial credit:

- At least one correct value
- Substitutes correctly at least once

High partial credit:

- 5 or 6 correct values

(b) Scale 15C (Accept candidates values from (a))

Low partial credit:

- At least 1 point plotted correctly

High partial credit:

- Points connected with straight edge
- At least 5 points plotted correctly and joined
- All points plotted correctly but not joined

Note: Answers to (c) (i) (ii) (iii) based on candidate's graph. Tolerance $\pm\frac{1}{2}$ unit

(c)(i) Scale 5B

Partial credit:

- Uses $t = 2.5$ to solve

(c)(ii) Scale 5B

Partial credit:

- Correct h ordinate indicated on graph

(c)(iii) Scale 5B

Partial credit:

- Only one ordinate given
- Coordinates reversed

(d)(i)(ii) Scale 10D

Low partial credit:

- Any work of merit e.g. correct formula given

Middle partial credit:

- One slope calculated correctly
- Both slopes calculated but with errors
- Correct answer to (ii) but no slope in (i)

High partial credit:

- Both slopes calculated correctly with incorrect or no conclusion

- (e)(i) Scale 5B
Partial credit:
- 1 term differentiated correctly

- (e)(ii) Scale 5C
Low partial credit:
- $\frac{dh}{dt} = 0$
 - Use of their $\frac{dh}{dt}$

- High partial credit:*
- The candidate's value of t substituted into expression for h
 - Solves their $\frac{dh}{dt} = 0$ correctly

- (e)(iii) Scale 5B
Partial credit:
- Recognition that speed = $\frac{dh}{dt}$
 - 3 substituted into $\frac{dh}{dt}$ and stops

- (f) Scale 5C
Low partial credit:
- Correct structure to $\frac{dh}{dt} = 2$
 - Correct answer with no work or without calculus

- High partial credit:*
- The candidate's value of t substituted into expression for h and stops or continues with errors
 - Solves their $\frac{dh}{dt} = 2$ correctly



Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014

Mathematics
(Project Maths – Phase 3)

Paper 2

Ordinary Level

Monday 9 June Morning 9:30 – 12:00

300 marks

Model Solutions – Paper 2

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her advising examiner.

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Questions 7 to 9.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1**(25 marks)**

A garage has 5 black cars, 9 red cars and 10 silver cars for sale.

(a) A car is selected at random. What is the probability that:

(i) The car is black?

$$P(\text{Black}) = \frac{5}{24}$$

(ii) The car is black or red?

$$P(\text{Black or Red}) = \frac{7}{12}$$

(b) A car is selected at random. Then a second car is selected at random from those remaining. What is the probability that:

(i) The first car is silver and the second car is black?

$$P(\text{First Silver and second Black}) = \frac{10}{24} \times \frac{5}{23} = \frac{25}{276}$$

(ii) One of the selected cars is red and the other is black?

$$P(\text{RB or BR}) = \frac{9}{24} \times \frac{5}{23} + \frac{5}{24} \times \frac{9}{23} = \frac{15}{92}$$

(c) Three of the black cars, two of the red cars and four of the silver cars have diesel engines. One car from the garage is again selected at random. What is the probability that it is a red car or a diesel car?

$$P(\text{Red or Diesel}) = \frac{9+9-2}{24} = \frac{2}{3}$$

Question 2**(25 marks)**

When taking a penalty kick, the probability that Kevin scores is always $\frac{3}{4}$.

- (a) Kevin takes a penalty. What is the probability that he does **not** score?

M= Kevin does not score

$$P(M) = \frac{1}{4}$$

- (b) Kevin takes two penalties. What is the probability that he scores both?

S= Kevin Scores

$$P(S,S) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

- (c) Kevin takes three penalties. What is the probability that he scores exactly twice?

$$P(S,S,M \text{ (in any order)}) = \binom{3}{2} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{27}{64}$$

- (d) Kevin takes five penalties. What is the probability that he scores for the first time on his fifth penalty?

$$P(M,M,M,M,S) = \left(\frac{1}{4}\right)^4 \times \frac{3}{4} = \frac{3}{1024}$$

Question 3**(25 marks)**

- (a) (i)**
- The circle
- c
- has equation
- $(x+2)^2 + (y-3)^2 = 100$
- .

Write down the co-ordinates of A , the centre of c .

$$A(-2, 3)$$

Write down r , the length of the radius of c .

$$r = \underline{\underline{10}}$$

- (ii)**
- Show that the point
- $P(-8, 11)$
- is on the circle
- c
- .

$$(-8+2)^2 + (11-3)^2 = 100$$

$$36 + 64 = 100$$

$$100 = 100$$

or

$$|AP| = \sqrt{(-8+2)^2 + (11-3)^2}$$

$$|AP| = \sqrt{100} = 10$$

- (b) (i)**
- Find the slope of the radius
- $[AP]$
- .

$$\text{Slope of radius} = -\frac{4}{3}$$

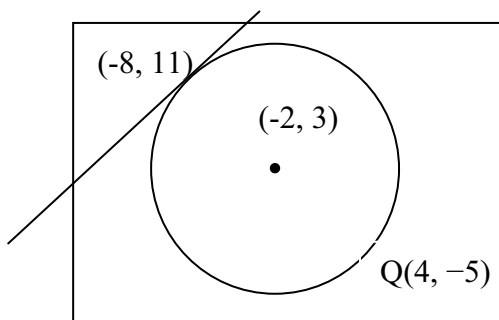
- (ii)**
- Hence, find the equation of
- t
- , the tangent to
- c
- at
- P
- .

$$\perp = \frac{3}{4}$$

$$y - 11 = \frac{3}{4}(x + 8)$$

$$3x - 4y + 68 = 0$$

- (c)**
- A second line
- k
- is a tangent to
- c
- at the point
- Q
- and
- $k \parallel t$
- . Find the co-ordinates of
- Q
- .

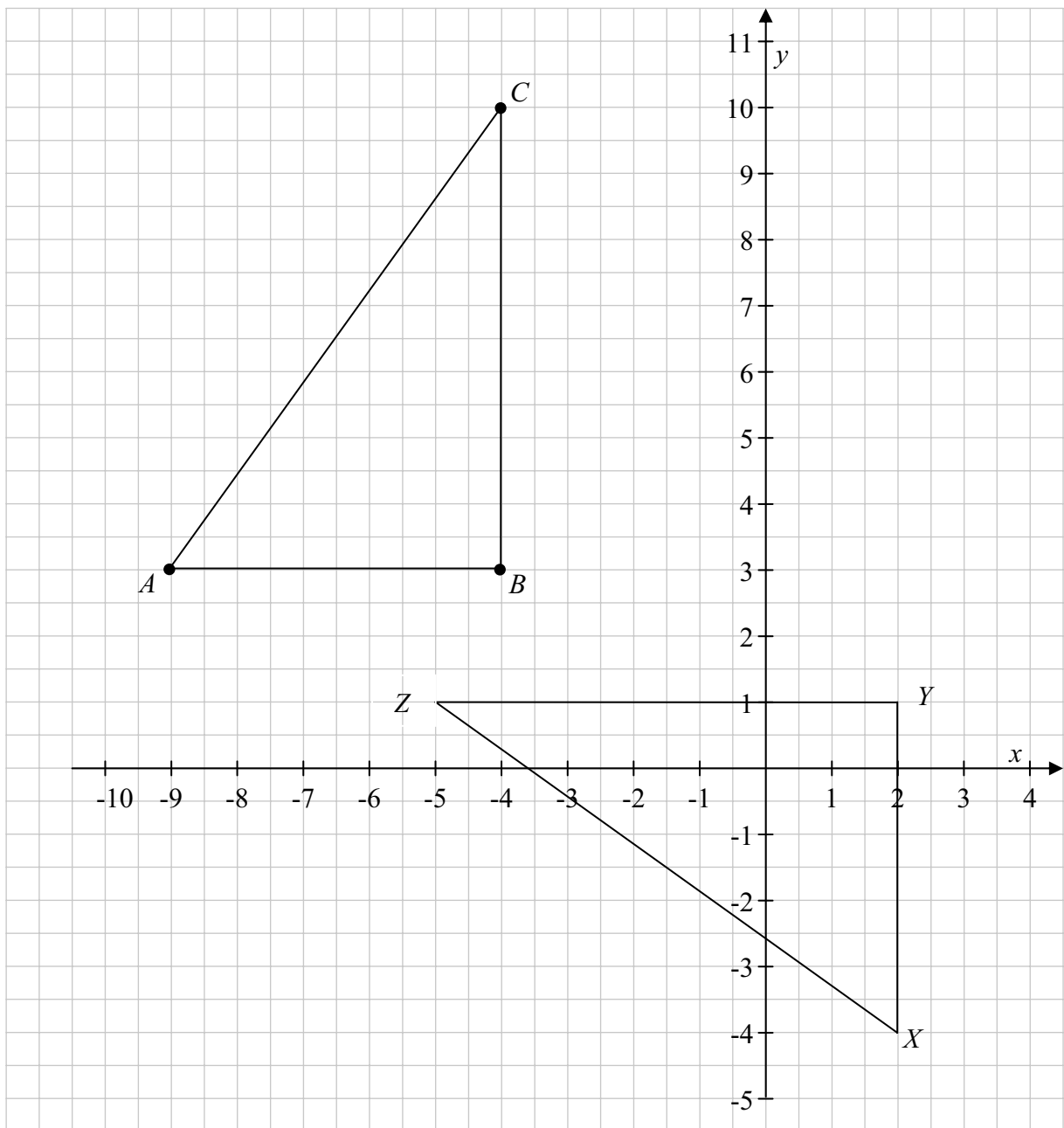


$$(-8, 11) \rightarrow (-2, 3) \quad x \uparrow 6, y \downarrow 8 \rightarrow Q(4, -5)$$

Question 4

(25 marks)

The points $A(-9, 3)$, $B(-4, 3)$ and $C(-4, 10)$ are the vertices of the triangle ABC , as shown.



- (a) (i) Find the length of $[AB]$.

$$\sqrt{(3-3)^2 + (-4+9)^2} = 5$$

or

$$5$$

(ii) Find the area of the triangle ABC .

$\begin{aligned} \text{Area} &= \frac{1}{2}(5)(7) \\ &= 17\frac{1}{2} \end{aligned}$	or	$\begin{aligned} &(-9, 3) \quad (-4, 3) \quad (-4, 10) \\ &x \uparrow 9, \quad y \downarrow 3 \\ &(0, 0) \quad (5, 0) \quad (5, 7) \\ \text{Area} &= \frac{1}{2}(5 \times 7 + 5 - 0) \\ &= 17\frac{1}{2} \end{aligned}$
--	-----------	---

(b) $X(2, -4)$ and $Y(2, 1)$ are two points.

(i) Draw, on the diagram above, a triangle, XYZ , which is congruent to the triangle ABC .

(ii) Write down the co-ordinates of Z and explain why the triangle XYZ is congruent to the triangle ABC .

$$Z = (\quad , \quad)$$

$Z = (-5, 1)$	or	$(-5, -4)$
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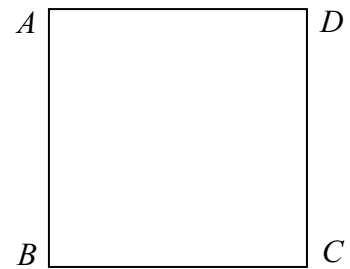
Reason:	SSS
	$ AB = XY \quad S$
	$ BC = YZ \quad S$
	$ AC = XZ \quad S$

Question 5

(25 marks)

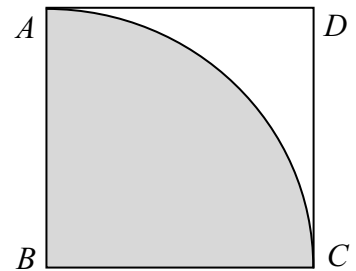
- (a) The square $ABCD$ has an area of 81 cm^2 . Find $|AD|$.

$$|AD| = \sqrt{81} = 9 \text{ cm}$$



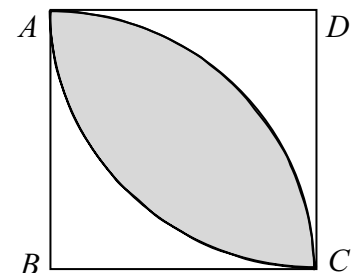
- (b) A sector of a circle, centre B and radius $|BC|$, is drawn inside $ABCD$ as shown by the shaded region.
 (i) Find the area of the sector, correct to one decimal place.

$$A = \frac{1}{4}(\pi \times 9^2) = 63.6 \text{ cm}^2$$



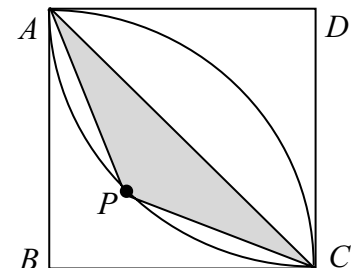
- (ii) A second sector of a circle, centre D and radius $|DA|$, is drawn. Find the area of the shaded region (the overlap of the two sectors), correct to one decimal place.

$$\begin{aligned} 81 - 63.61 &= 17.4 \\ \text{Shaded area} &= 81 - 2(17.4) \\ &= 46.2 \text{ cm}^2 \end{aligned}$$



- (c) The point P is on the arc of the sector DAC , as shown. The triangle APC is isosceles. Find the area of the triangle APC , correct to one decimal place.

$$\begin{aligned} |AC| &= \sqrt{162} = 9\sqrt{2} \\ |PO| &= 9 - \frac{1}{2}(9\sqrt{2}) = \frac{18 - 9\sqrt{2}}{2} \\ APC &= \frac{1}{2}(9\sqrt{2})\left(\frac{18 - 9\sqrt{2}}{2}\right) \\ APC &= \frac{81\sqrt{2} - 81}{2} = 16.8 \text{ cm}^2 \end{aligned}$$



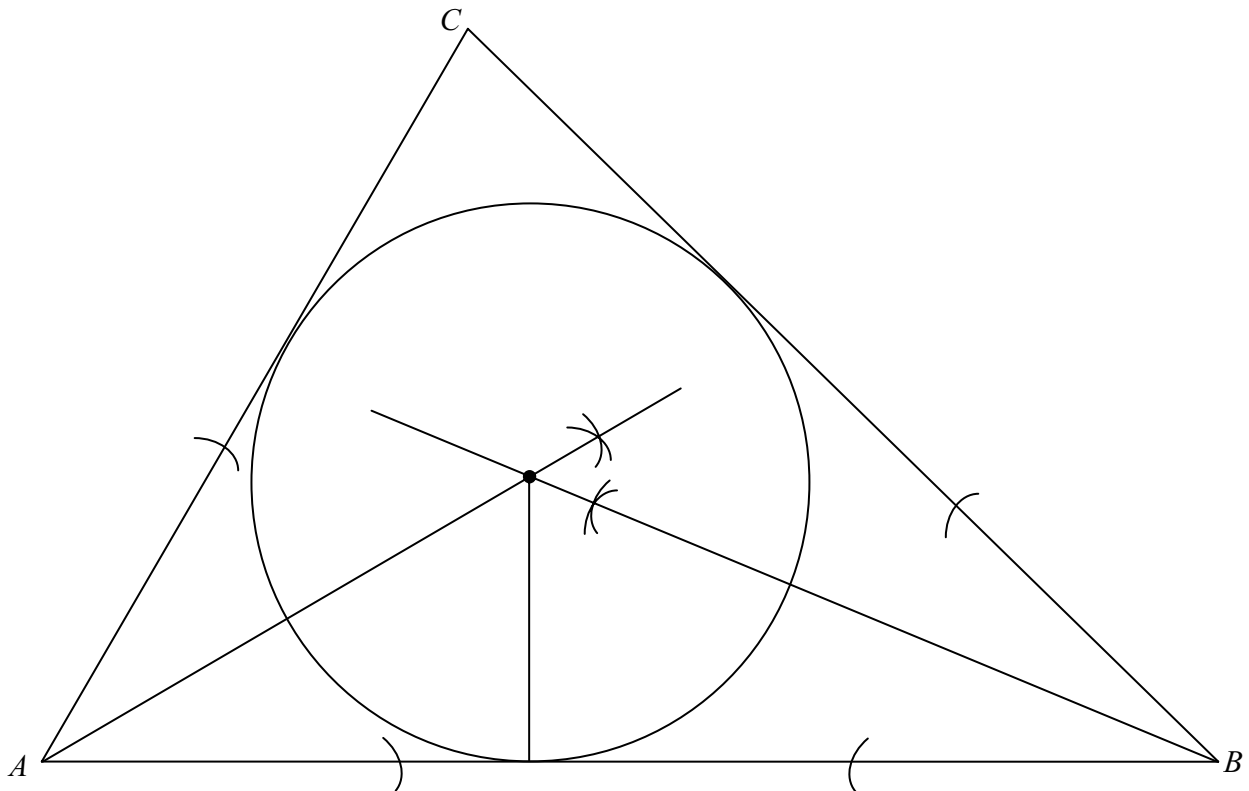
Question 6

(25 marks)

Answer **either** 6A **or** 6B.

Question 6A

- (a) (i) Construct the incircle of the triangle ABC below. Show all your construction lines clearly.



- (ii) Measure the length of the radius of the circle constructed in part (i).

Length of radius:

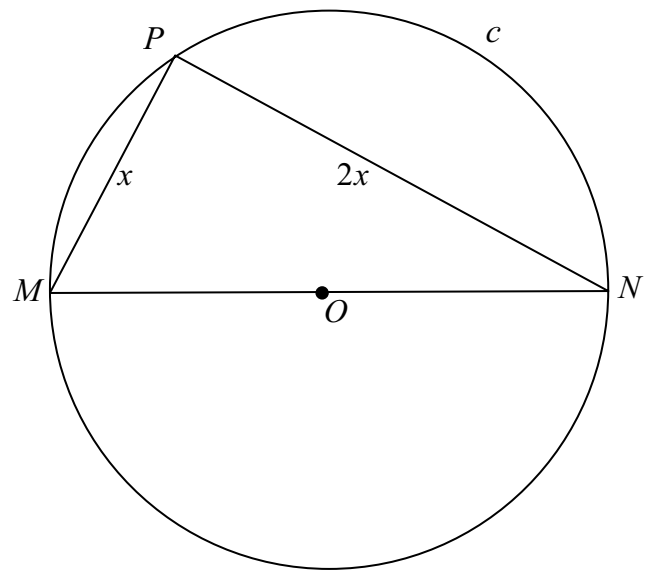
3.6 cm

(b) The point P is on the circle c with centre O and diameter $[MN]$, as shown.

The length of the radius of c is $2\sqrt{5}$ cm.

$|MP| = x$ cm and $|PN| = 2x$ cm.

Find the value of x .



$$(4\sqrt{5})^2 = x^2 + (2x)^2$$

$$16(5) = x^2 + 4x^2$$

$$80 = 5x^2 \Rightarrow x^2 = 16$$

$$x = 4 \text{ cm}$$

OR

Question 6B

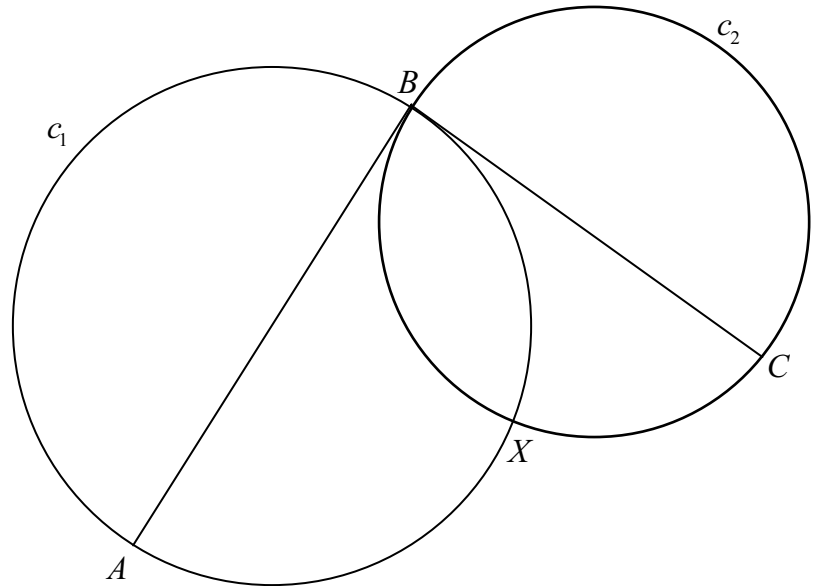
Two circles, c_1 and c_2 , intersect at the points B and X , as shown.
The circle c_1 has diameter $[AB]$.

The circle c_2 has diameter $[BC]$.

The line CB is a tangent to c_1 .

The line CB is a tangent to c_1 .

Prove that X is on the line AC .



Join B to X

$[AB]$ is a diameter of $c_1 \Rightarrow \angle AXB = 90^\circ$ (Angle in a semicircle)

$[BC]$ is a diameter of $c_2, \Rightarrow \angle CXB = 90^\circ$ (Angle in a semicircle)

$\Rightarrow \angle AXC = 180^\circ$

$\Rightarrow A, X,$ and C are collinear

Answer **all three** questions from this section.

Question 7**(70 marks)**

A newspaper report in October 2013 stated that 90% of homeowners who were liable for property tax had registered for it. The total number of properties liable for the tax was estimated at 1.9 million.

- (a) (i) Estimate the number of properties that were registered.

$$1.9 \times 0.9 = 1.71$$

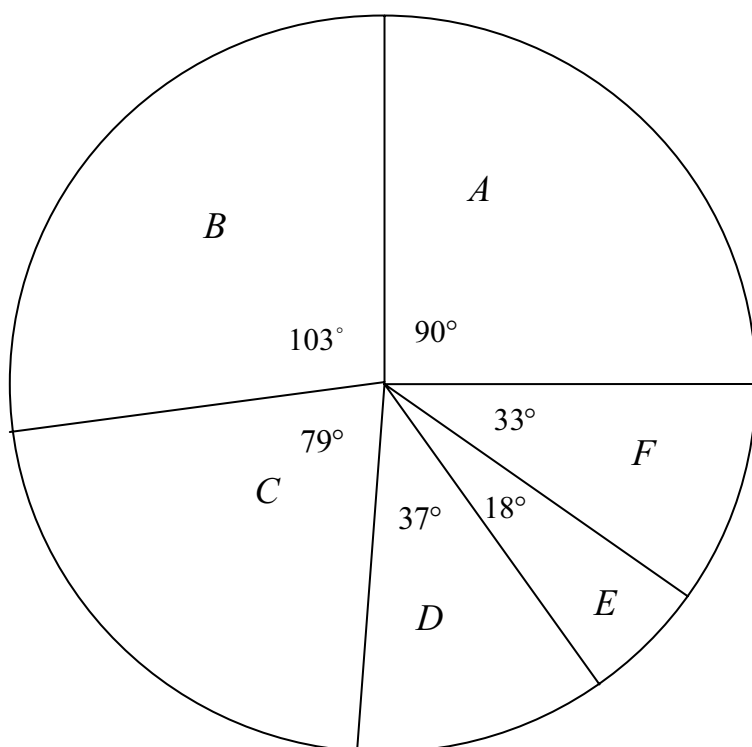
- (ii) Suggest one reason why some properties were not registered.

- Owner might be abroad
- Owner unable to do paperwork
- Owner unable to pay
- Trying to avoid tax

- (b) Homeowners, who registered, were required to value their property in one of a number of given Valuation Bands. The percentage who had valued their properties in each Valuation Band is given in the table below.

Valuation Band	€0 - €100 000	€100 001 - €150 000	€150 001 - €200 000	€200 001 - €250 000	€250 001 - €300 000	Over €300 000
Percentage of registered homeowners	24.9	28.6	21.9	10.4	4.9	9.3

Represent the data in the table using the pie chart below. Label each sector you create and show the angle in each sector clearly.



$$\begin{aligned}
 A & \frac{24.9}{100} \times 360 = 89.64^\circ \approx 90^\circ \\
 B & \frac{28.6}{100} \times 360 = 102.96^\circ \approx 103^\circ \\
 C & \frac{21.9}{100} \times 360 = 78.84^\circ \approx 79^\circ \\
 D & \frac{10.4}{100} \times 360 = 37.44^\circ \approx 37^\circ \\
 E & \frac{4.9}{100} \times 360 = 17.64^\circ \approx 18^\circ \\
 F & \frac{9.3}{100} \times 360 = 33.48^\circ \approx 33^\circ
 \end{aligned}$$

(c) (i) Use the data in the table above and your answer to part **(a) (i)** above to complete the following table.

Valuation Band	Tax per property	Number of properties	Total tax due (€)
€0 - €100 000	€45	425 790	19 160 550
€100 001 - €150 000	€112	489 060	54 774 720
€150 001 - €200 000	€157	374 490	58 794 930
€200 001 - €250 000	€202	177 840	35 923 680
€250 001 - €300 000	€247	83 790	20 696 130
Over €300 000	NA	159 030	NA

NA = Not Available

- (ii) Find the total tax due on those properties, registered by October 2013, with a valuation up to €300 000

$$\text{Total (up to €300 000)} = €189\,350\,010$$

- (iii) The total tax due on all the properties that were registered was estimated at €241 million. Find the total tax due on those properties with a valuation over €300 000.

$$241\,000\,000 - 189\,350\,010 = €51\,649\,990$$

- (iv) Find the mean estimated tax per property on those properties with a valuation over €300 000.

$$\frac{51\,649\,990}{159\,030} = €324.78$$

- (v) Some homeowners may under-value their property in order to pay less tax. For example, one estimate stated that 20% of properties in the €100 001 - €150 000 band should have been valued in the €150 001 - €200 000 band. Based on this estimate, find the amount of extra tax that would be raised if these properties were registered in the correct Valuation Band.

$$\begin{aligned} 20\% \times 489\,060 &= 97\,812 \\ 97\,812(157 - 112) &= €4\,401\,540 \end{aligned}$$

Question 8**(50 marks)**

(a) A wind turbine, used to generate electricity, has three equally spaced blades 65 metres long.

- (i) Write down the size of the angle between two blades.

120° (240°)



- (ii) Find the area of the disc traced out by one full rotation of the blades, correct to the nearest whole number.

$$\text{Area} = \pi \times 65^2 = 13273 \text{ m}^2 \quad (13267 \text{ m}^2) \quad (13279 \text{ m}^2)$$

- (iii) Find the area of the triangle formed by joining the tips of the three blades, correct to the nearest whole number.

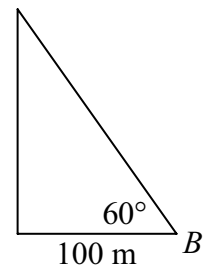
$$\text{Area:} \quad 3 \left(\frac{1}{2} \right) (65)(65) \sin 120^\circ = 5488 \text{ m}^2$$

- (iv) The expected lifetime of the turbine is 25 years. On average, the turbine operates 31% of the time. The blades rotate 15 times per minute when the turbine is operating. Find the number of times the blades will rotate during the expected lifetime of the turbine (ignore leap years). Write your answer in the form $a \times 10^n$, where $1 \leq a < 10$ and $n \in \mathbb{Z}$.

$$\begin{aligned} & 15 \times 60 \times 24 \times 365 \times 25 \times 0.31 \\ &= 61101000 \\ &= 6.1101 \times 10^7 \end{aligned}$$

- (b) Gráinne stood at a point B , which is on level ground 100 metres from the base of the tower supporting the blades, as shown. From there, she measured the angle of elevation to the top of the tower as 60° . Find the height of the tower, using Gráinne's measurements. Give your answer correct to the nearest metre.

$$\begin{aligned} \tan 60^\circ &= \frac{h}{100} \\ h &= 100 \tan 60^\circ \\ h &= 173 \text{ m} \end{aligned}$$



(c) Gráinne recognises that her measurement of the angle may not be totally accurate. She read elsewhere that the actual height of the tower is 154 m.

(i) If Gráinne measured the 100 m accurately, find the actual size of the angle at B , correct to the nearest degree.

$$\tan B = \frac{154}{100}$$

$$B = \tan^{-1} \frac{154}{100}$$

$$B = 57^\circ$$

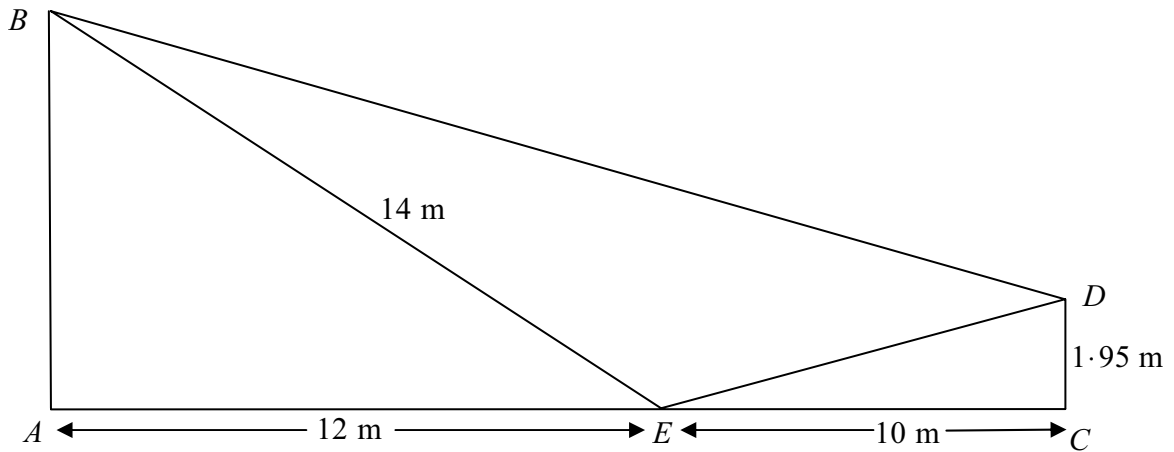
(ii) Find the percentage error in Grainne's measurement of the angle of elevation, correct to one decimal place.

$$\begin{aligned} \text{Percentage error} &= \frac{\text{Error}}{\text{True Value}} \times 100 \\ &= \frac{3}{57} \times 100 \\ &= 5.3\% \end{aligned}$$

Question 9**(30 marks)**

At an activity centre a zip-line, $[BD]$, runs between two vertical poles, $[AB]$ and $[CD]$, on level ground, as shown. The point E is on the ground, directly below the zip-line.

$|AE| = 12$ m, $|BE| = 14$ m, $|CD| = 1.95$ m, and $|EC| = 10$ m.



- (a) (i) Find the distance $|ED|$, correct to one decimal place.

$$|ED| = \sqrt{10^2 + 1.95^2} = 10.2 \text{ m} \quad (\sqrt{103.8025}) (10.18)$$

- (ii) Find $|\angle AEB|$, correct to the nearest degree.

$$\begin{aligned} \cos \angle AEB &= \frac{12}{14} \\ |\angle AEB| &= \cos^{-1} \frac{12}{14} \\ |\angle AEB| &= 31^\circ \end{aligned}$$

- (b) (i) Find $|\angle DEB|$, given that $|\angle CED| = 11^\circ$, correct to the nearest degree.

$$|\angle DEB| = 180 - (11 + 31)$$

$$|\angle DEB| = 180 - 42$$

$$|\angle DEB| = 138^\circ$$

- (ii) Hence, or otherwise, find the distance $|DB|$.
Give your answer correct to one decimal place.

$$|DB|^2 = 14^2 + 10 \cdot 2^2 - 2(14)(10 \cdot 2)\cos 138^\circ$$

$$|DB|^2 = 512 \cdot 28216$$

$$|DB| = 22 \cdot 6336$$

$$|DB| = 22 \cdot 6 \text{ m}$$

or

$$\tan 31^\circ = \frac{|AB|}{12} \Rightarrow |AB| = 7 \cdot 210$$

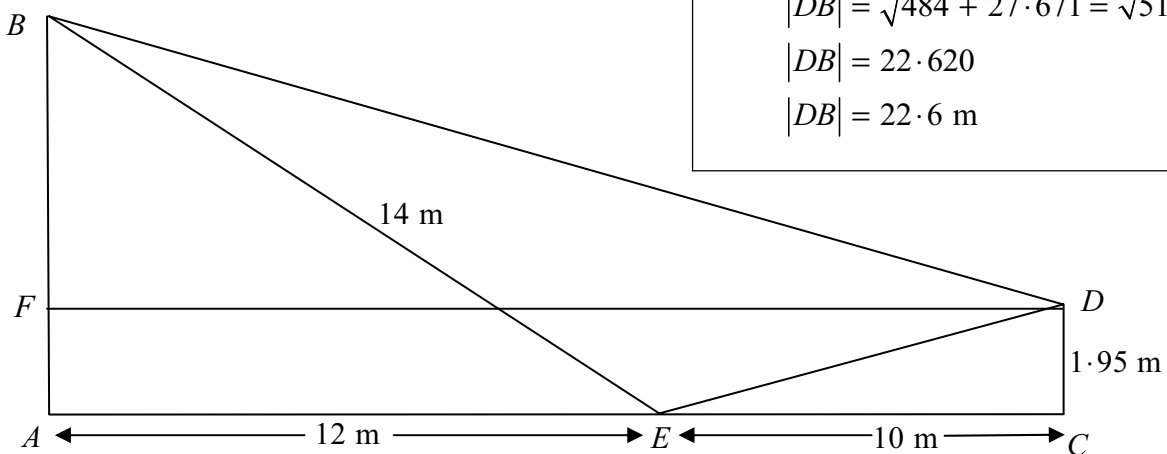
$$|FB| = 7 \cdot 21 - 1 \cdot 95 = 5 \cdot 260 \dots$$

$$|DB| = \sqrt{(10 + 12)^2 + (5 \cdot 260 \dots)^2}$$

$$|DB| = \sqrt{484 + 27 \cdot 671} = \sqrt{511 \cdot 671}$$

$$|DB| = 22 \cdot 620$$

$$|DB| = 22 \cdot 6 \text{ m}$$



Marking Scheme – Paper 2, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 3, 5	0, 3, 4, 5		
10 mark scales		0, 5, 10	0, 4, 7, 10		
15 mark scales		0, 7, 15			
20 mark scales		0, 10, 20			
25 mark scales					0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does oversimplify the work a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, *scale 10C*, 9 marks may be awarded.

- Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1	
(a)(i)(ii)	10C
(b)(i)	5C
(b)(ii)	5C
(c)	5C
Question 2	
(a)	10B
(b)	5C
(c)	5A
(d)	5C
Question 3	
(a)(i)(ii)	5C
(b)(i)	10C
(b)(ii)	5C
(c)	5C
Question 4	
(a)(i)	10B
(a)(ii)	10C
(b)(i)(ii)	5C
Question 5	
(a)	5B
(b)(i)	10C
(b)(ii)	5B
(c)	5A
Question 6A	
(a)(i)	15B
(a)(ii)	5B
(b)	5B
Question 6B	
	25E

Section B

Question 7	
(a)(i)(ii)	10C
(b) angles	5C
(b) pie	20B
(c)(i) number	5C
(c)(i) tax	10C
(c)(ii)	5B
(c)(iii)	5B
(c)(iv)	5B
(c)(v)	5B
Question 8	
(a)(i)	10B
(a)(ii)	10C
(a)(iii)	5C
(a)(iv)	10B
(b)	5C
(c)(i)	5C
(c)(ii)	5C
Question 9	
(a)(i)	10C
(a)(ii)	5C
(b)(i)	10C
(b)(ii)	5C

Section A

Question 1

(a)(i)(ii) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Relevant work in one or both sections. e.g. Either #E or #S correct.

High partial credit:

- One correct section.

Full credit:

- Accept correct answers without work.

(b)(i) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Any correct #E or #S.
- $P(\text{silver}) = \frac{10}{24}$ and/or $P(\text{black}) = \frac{5}{23}$

High partial credit:

- $(\frac{10}{24} \times \frac{5}{23})$ and stops.
- Calculates the probability with replacement. $(\frac{10}{24} \times \frac{5}{24} = \frac{50}{576}$ or $\frac{25}{288}$)
- Correct answer without work.

(b)(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Any relevant work towards creating at least one correct probability.

High partial credit:

- $[(\frac{9}{24} \times \frac{5}{23}) + (\frac{5}{24} \times \frac{9}{23})]$ and stops.
- Correct answer without work.

(c) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Any relevant work towards creating at least one correct probability.
- Solution presented as. $[P(\text{red or diesel}) = \frac{9}{24} + \frac{9}{24} = \frac{18}{24}$ or $\frac{3}{4}]$

High partial credit:

- $(\frac{9}{24} + \frac{9}{24} - \frac{2}{24})$
- Correct answer without work.

Question 2

(a) Scale 10B (0, 5, 10)

Partial credit:

- Writes the correct #E or #S.
- Any incomplete work with 1.

Full credit:

- Accept correct answers without work.

(b) Scale 5C (0, 3, 4, 5).

Low partial credit:

- Identifies the probability of scoring here.
- Writes $P(S, S)$ as $(\frac{3}{4} + \frac{3}{4})$

High partial credit:

- Fails to finish. $[P(S, S) = (\frac{3}{4} \times \frac{3}{4})]$ or $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Full credit:

- Accept correct answers without work.

(c) Scale 5A (0, 5)

Full credit:

- Accept correct answers without work.

(d) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Correctly identifies $P(S)$ and/or $P(M)$.
- Lists the outcomes as M,M,M,M,S, or writes as $(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{3}{4})$

High partial credit:

- A fully correct substitution. i.e. $(\frac{1}{4})^4 \times \frac{3}{4}$

Full credit:

- Accept correct answers without work.

Question 3

(a)(i)(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Relevant work in one section.
e.g. Finds one correct co-ordinate of the centre A or works to $r^2 = 100$ and/or $\sqrt{100}$ and stops.

High partial credit:

- One section correct.

(b)(i) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Identifies correct relevant formula, e.g. writes $(m = \frac{y_2 - y_1}{x_2 - x_1})$
- Any correct or consistent substitution.

High partial credit:

- Formula substituted correctly
- Calculates slope with one incorrect substitution.

Full credit:

- Accept for full credit answer as $\frac{8}{-6}$ without work.

(b)(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Identifies correct relevant formula, e.g. writes $y - y_1 = m(x - x_1)$.
- Correct or consistent slope identified in this part and stops.

High partial credit:

- A fully correct or consistent substitution. e.g. $[y - 11 = \frac{3}{4}(x + 8)]$.
- Calculates equation of line with one incorrect substitution.

(c) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Writes slope of k equal to slope of t .
- Identifies correct relevant formula, e.g. writes $[x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2}]$.
- Work with a translation which indicates an understanding of the concept.
- Any work of merit using algebraic method.

High partial credit:

- An incorrect translation correctly applied.
- A correct graphical solution.
- Finds both points of intersection algebraically and stops.

Question 4

(a)(i) Scale 10B (0, 5, 10)

Partial credit:

- Identifies correct relevant formula, e.g. writes $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and stops.
- Any correct substitution into formula.
- Calculates a distance with one incorrect substitution.
- Incorrect reading of the scale diagram. (Distance = 10) i.e. Counting Boxes.

Full credit:

- Accept correct answer without work.

(a)(ii) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Writes a correct formula for the area of the triangle
- Any correct work towards the area of triangle calculation.

High partial credit:

- A fully correct substitution.
- Calculates area with one incorrect substitution.

Full credit:

- Accept correct answer without work.

(b)(i)(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Relevant work in one section. e.g. Plots one or both points correctly.
- Work of merit in at least one section.

High partial credit:

- One section correct.

Question 5

(a) Scale 5B (0, 3, 5)

Partial credit:

- Works to $x^2 = 81$ or $x = \sqrt{81}$ and stops.

Note: A correct answer without work shown award full credit.

(b)(i) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Writes the correct formula for the sector area or area of circle.
- Any correct or consistent substitution into formula.
- Any work of merit e.g. $r = 9$ or Answer (a).

High partial credit:

- Formula substituted correctly.
- Calculates area of sector with one incorrect substitution.
- Correct or consistent answer expressed in terms of π .

Full credit:

- Accept correct answers without work.

(b)(ii) Scale 5B (0, 3, 5)

Partial credit:

- Calculates a relevant area. e.g. Unshaded area worked as $(81 - 63 \cdot 64)$ or similar and stops.
- Concept of shaded area by difference indicated but not fully developed.

Note: Accept an answer consistent with previous work.

(c) Scale 5A (0, 5)

Full credit:

- Correct answer.

Note: Accept an answer consistent with previous work.

Question 6A

(a)(i) Scale 15B (0, 7, 15)

Partial credit:

- Any work of merit e.g. One arc drawn from a vertex.

Full credit:

- An incircle constructed with at least one side of the triangle a tangent to the circle

(a)(ii) Scale 5B (0, 3, 5)

Partial credit:

- Accept length of radius in intervals $3 \cdot 3 \leq r < 3 \cdot 5$ and $3 \cdot 7 < r \leq 3 \cdot 9$.

Full credit:

- Award full credit to radius in interval $3 \cdot 5 \leq r \leq 3 \cdot 7$.

(b) Scale 5B (0, 3, 5)

Partial credit:

- Diameter calculated as $4\sqrt{5}$ or similar.
- Any relevant work in the use of Pythagoras or recognition of angle standing on diameter at circumference measures 90° .

Question 6B

Scale 25E (0, 5, 10, 15, 20, 25).

Low partial credit:

- Any work of merit such as joining B to X .

Lower middle partial credit:

- Identifies diameter of c_1 or c_2 , and one angle standing on semi-circle at circumference measuring 90°

Upper middle partial credit

- Identifies diameter of c_1 and c_2 , and both angles standing on semi-circle at circumference measuring 90°

High partial credit:

- Identifies $|\angle AXC| = 180^\circ$

Section B

Question 7

(a)(i)(ii) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Any work of merit towards estimating the number of properties registered or suggesting a partially plausible reason.
e.g. 90% expressed as 0.9.

High partial credit:

- One section correct.

(b) Angle Determination

Scale 5C (0, 3, 4, 5)

Low partial credit:

- One sector angle correct
- More than one error in the calculation of sector angle.
- Any work of merit. e.g. Correct work with percentage or use of 360.

High partial credit:

- One consistent error in the calculation of sector angle.

Pie Chart

Scale 20B (0, 10, 20)

Partial credit:

- Any one sector indicated.

(c)(i) Number of Properties.

Scale 5C (0, 3, 4, 5)

Low partial credit:

- One correct entry.

High partial credit:

- One consistent error in the calculation.

Total Tax due.

Scale 10C (0, 4, 7, 10)

Low partial credit:

- One correct entry.

High partial credit:

- One consistent error in the calculation.

(c)(ii) Scale 5B (0, 3, 5)

Partial credit:

- Some addition of the correct values indicated. e.g. (19 160 550+54 774 720+....)

Full credit:

- Accept a correct or consistent answer without work.

(c)(iii) Scale 5B (0, 3, 5)

Partial credit:

- Writes 241,000,000 or Answer c(ii).

Full credit:

- Accept a correct or consistent answer without work.

(c)(iv) Scale 5B (0, 3, 5)

Partial credit:

- Work of merit with any of the values [1 550 970, 159 030, 51 649 990].

(c)(v) Scale 5B (0, 3, 5)

Partial credit:

- Any work of merit in finding the number of properties changing band e.g. $((0.2)(489060))$
- Identifies the difference in the band valuation. i.e. $(157 - 112)$
- Fully correct substitution. i.e. $[(0.2)(489060) \times 45]$

Question 8

(a)(i) Scale 10B (0, 5, 10)

Partial credit:

- Any work of merit. e.g. Joins tips of blades or draws circle on diagram.

Full credit:

- Accept a correct answer without work. (240°).

(a)(ii) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Writes the correct formula.
- Identifies the radius correctly. ($r = 65$).

High partial credit:

- A fully correct substitution.
- Calculates the area of circle with one incorrect substitution.
- Answer left in terms of π .

Full credit:

- Accept a correct answer without work.

(a)(iii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Writes a correct formula for area of a triangle.
- Some correct substitution into a correct relevant formula. [Radius = 65, $\theta = 120^\circ$]
- Any work of merit such as a triangle drawn in this section with the tips of blades joined.

High partial credit:

- Fully correct or consistent substitution. e.g. $3\left(\frac{1}{2}65^2 \sin 120^\circ\right)$
- Calculates the area of the triangle with one incorrect substitution.
- Incorrect calculator mode. (Once only). [Rad = 3680, Grad = 6027].

Full credit:

- Accept a correct answer without work.

(a)(iv) Scale 10B(0, 5,10)

Partial credit:

- Any work of merit in the determination of working life or number of rotations.
e.g. $25(0.31)$ or $365(24)(60)15$

Full credit:

- A correct answer.

(b) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Any correct relevant work such as a trigonometric ratio or the Sine Rule identified.
- Any correct additional information identified in given triangle.

High partial credit:

- Correct substitution. i.e. [$\tan 60^\circ = \frac{h}{100}$]
- Calculates the height with one incorrect substitution.

Full credit:

- Accept a correct answer without work.

(c)(i) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Any work of merit involving Pythagoras Theorem, Tan ratio or identification of a right angled triangle.

High partial credit:

- Correct substitution into formula or $|\angle B| = \tan^{-1}\left(\frac{154}{100}\right)$.
- Calculates $|\angle B|$ with one incorrect substitution.

Full credit:

- Accept a correct answer without work.

(c)(ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Any work of merit. e.g. Calculates the size of the error or writes % error formula.

High partial credit:

- A fully correct substitution. $(\frac{3}{57} \times 100)$.
- Calculates the % error with one incorrect substitution.

Question 9

(a) (i) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Any relevant work. e.g. A correct relevant formula written e.g. Pythagoras, Cosine Rule, Sine Rule or part calculation $(10)^2 = 100$

Note: Accept use of information from (b) (i) in this part i.e. $|\angle CED| = 11^\circ$.

High partial credit:

- A fully correct substitution.
- Calculates $|ED|$ with one error in substitution.

Full credit:

- Accept a correct answer without work.

(a) (ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Indicates correct relevant information on the diagram.
- Correct trigonometric ratio set up or similar work of merit.

High partial credit:

- A fully correct substitution.
- Calculates $|\angle AEB|$ with one error in substitution.

Full credit:

- Accept a correct answer without work.

(b) (i) Scale 10C (0, 4, 7, 10)

Low partial credit:

- Indicates correct or consistent relevant information on the diagram.
- e.g. $|\angle CED| = 11^\circ$, $|\angle AEB| = 31^\circ$
- Any work of merit. e.g. Total angle measure in a straight line = 180° .

High partial credit

- A fully correct or consistent substitution. i.e. $180^\circ - (11^\circ + 31^\circ)$.
- Calculates $|\angle DEB|$ with one error in substitution

Note: Accept an answer consistent with previous work. [See (a)(ii)]

(b) (ii) Scale 5C (0, 3, 4, 5)

Low partial credit:

- Indicates correct or consistent relevant information on the diagram.
- Identification or use of relevant trigonometric formula. e.g. Cosine Rule.

High partial credit:

- A fully correct or consistent substitution. i.e. $|DB^2| = 14^2 + 10 \cdot 2^2 - 2(14)(10 \cdot 2)\cos 138^\circ$
- Calculates $|BD|$ with one error in substitution.
- Incorrect calculator mode.(once only)

Note: Accept an answer consistent with previous work.

Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. $198 \text{ marc} \times 5\% = 9.9 \Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle $[300 - \text{bunmharc}] \times 15\%$, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

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