Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2014

Mathematics
(Project Maths – Phase 3)

Paper 2

Ordinary Level

Monday 9 June   Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions, as follows:

In Section A, answer:

> Questions 1 to 5 and
>
> **either** Question 6A **or** Question 6B.

In Section B, answer Questions 7 to 9.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

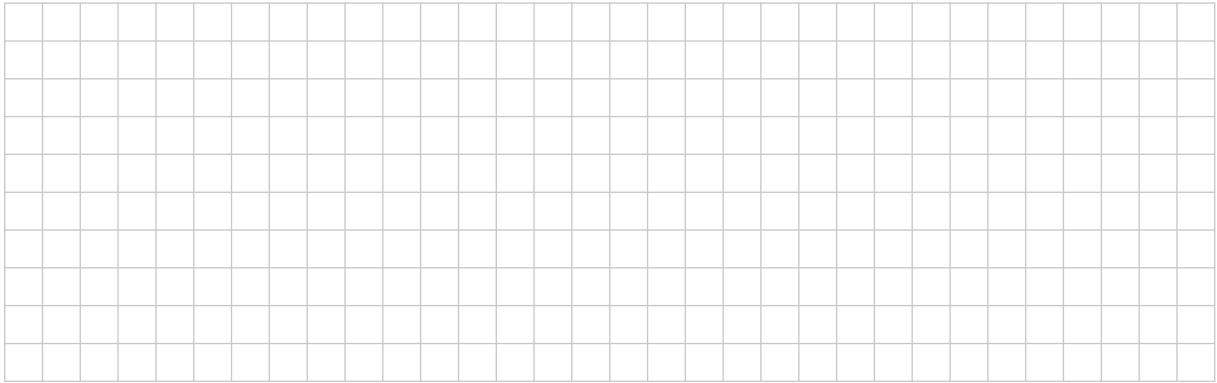Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1   **(25 marks)**

A garage has 5 black cars, 9 red cars and 10 silver cars for sale.

(a) A car is selected at random. What is the probability that:

     (i) The car is black?

     (ii) The car is black or red?

(b) A car is selected at random. Then a second car is selected at random from those remaining. What is the probability that:

     (i) The first car is silver and the second car is black?

     (ii) One of the selected cars is red and the other is black?

(c) Three of the black cars, two of the red cars and four of the silver cars have diesel engines. One car from the garage is again selected at random. What is the probability that it is a red car or a diesel car?

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Question 2 (25 marks)

When taking a penalty kick, the probability that Kevin scores is always $\frac{3}{4}$.

(a) Kevin takes a penalty. What is the probability that he does **not** score?

(b) Kevin takes two penalties. What is the probability that he scores both?

(c) Kevin takes three penalties. What is the probability that he scores exactly twice?

(d) Kevin takes five penalties. What is the probability that he scores for the first time on his fifth penalty?

Question 3 **(25 marks)**

(a) **(i)** The circle c has equation $(x+2)^2 + (y-3)^2 = 100$.

Write down the co-ordinates of A, the centre of c. $A\,(\quad,\quad)$

Write down r, the length of the radius of c. $r = \underline{\qquad}$

(ii) Show that the point $P(-8,\ 11)$ is on the circle c.

(b) **(i)** Find the slope of the radius $[AP]$.

(ii) Hence, find the equation of t, the tangent to c at P.

(c) A second line k is a tangent to c at the point Q and $k \parallel t$. Find the co-ordinates of Q.

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Question 4 **(25 marks)**

The points $A(-9, 3)$, $B(-4, 3)$ and $C(-4, 10)$ are the vertices of the triangle ABC, as shown.

(a) **(i)** Find the length of $[AB]$.

(ii) Find the area of the triangle *ABC*.

(b) *X*(2, −4) and *Y*(2, 1) are two points.

(i) Draw, on the diagram above, a triangle, *XYZ*, which is congruent to the triangle *ABC*.

(ii) Write down the co-ordinates of *Z* and explain why the triangle *XYZ* is congruent to the triangle *ABC*.

$Z = ($ $,$ $)$

Reason:

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Question 5 (25 marks)

**(a)** The square *ABCD* has an area of 81 cm². Find | *AD* |.
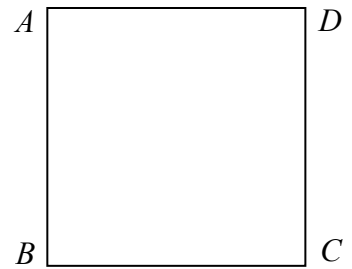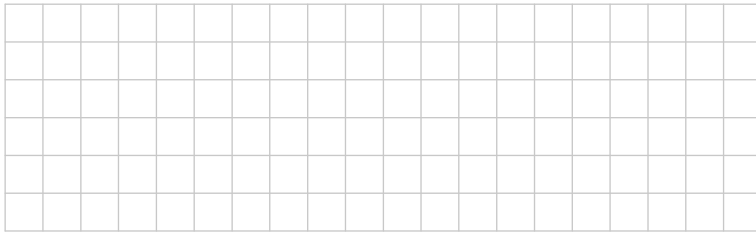
(b) A sector of a circle, centre *B* and radius | *BC* |, is drawn
inside *ABCD* as shown by the shaded region.
   (i) Find the area of the sector, correct to one decimal place.
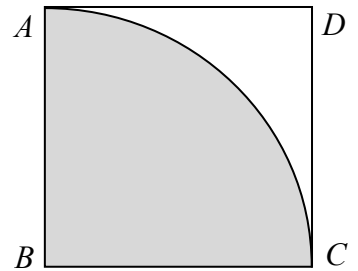
   (ii) A second sector of a circle, centre *D* and radius | *DA* |,
   is drawn. Find the area of the shaded region (the
   overlap of the two sectors), correct to one decimal
   place.
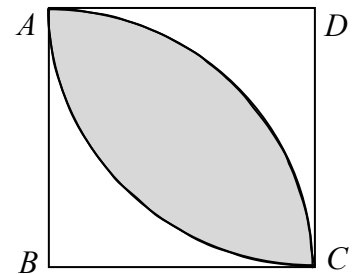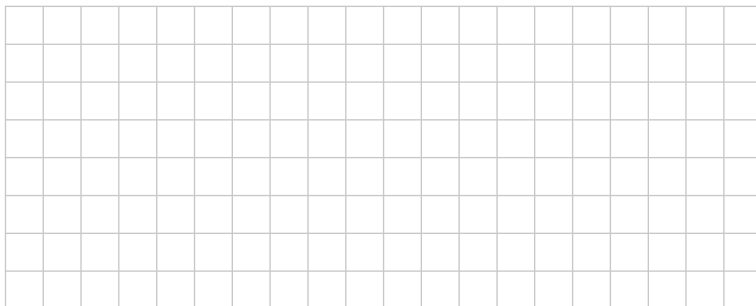
(c) The point *P* is on the arc of the sector *DAC*, as shown. The
triangle *APC* is isosceles. Find the area of the triangle *APC*,
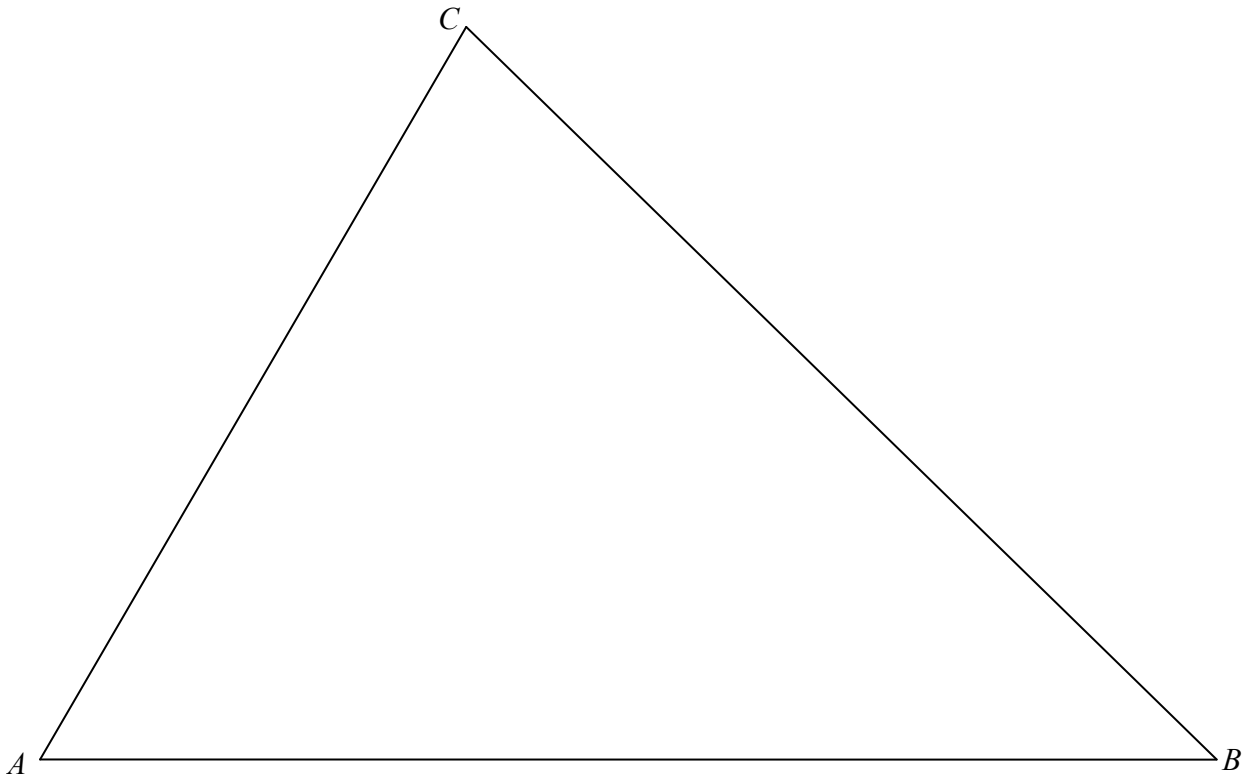correct to one decimal place.
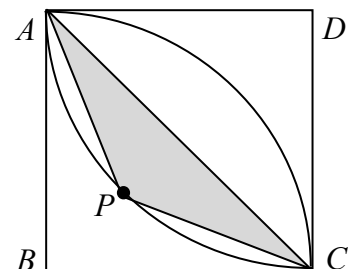
Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) **(i)** Construct the incircle of the triangle *ABC* below. Show all your construction lines clearly.

(ii) Measure the length of the radius of the circle constructed in part **(i)**.

Length of radius: _____

page | running

(b) The point P is on the circle c with centre O and diameter $[MN]$, as shown.

The length of the radius of c is $2\sqrt{5}$ cm.

$|MP| = x$ cm and $|PN| = 2x$ cm.

Find the value of x.

OR

Question 6B

Two circles, c_1 and c_2, intersect at the points B and X, as shown.
The circle c_1 has diameter $[AB]$.
The circle c_2 has diameter $[BC]$.
The line CB is a tangent to c_1.

Prove that X is on the line AC.

Answer **all three** questions from this section.

Question 7 **(70 marks)**

A newspaper report in October 2013 stated that 90% of homeowners who were liable for property tax had registered for it. The total number of properties liable for the tax was estimated at 1·9 million.

**(a)** **(i)** Estimate the number of properties that were registered.

(ii) Suggest one reason why some properties were not registered.

(b) Homeowners, who registered, were required to value their property in one of a number of given Valuation Bands. The percentage who had valued their properties in each Valuation Band is given in the table below.

Valuation Band	€0 - €100 000	€100 001 - €150 000	€150 001 - €200 000	€200 001 - €250 000	€250 001 - €300 000	Over €300 000
Percentage of registered homeowners	24·9	28·6	21·9	10·4	4·9	9·3

Represent the data in the table using the pie chart below. Label each sector you create and show the angle in each sector clearly.

(c) **(i)** Use the data in the table above and your answer to part **(a) (i)** above to complete the following table.

Valuation Band	Tax per property	Number of properties	Total tax due (€)
€0 - €100 000	€45	425 790	19 160 550
€100 001 - €150 000	€112	489 060	
€150 001 - €200 000	€157		
€200 001 - €250 000	€202		
€250 001 - €300 000	€247		
Over €300 000	NA		NA

NA = Not Available

(ii) Find the total tax due on those properties, registered by October 2013, with a valuation up to €300 000.

(iii) The total tax due on all the properties that were registered was estimated at €241 million. Find the total tax due on those properties with a valuation over €300 000.

(iv) Find the mean estimated tax per property on those properties with a valuation over €300 000.

(v) Some homeowners may under-value their property in order to pay less tax. For example, one estimate stated that 20% of properties in the €100 001 - €150 000 band should have been valued in the €150 001 - €200 000 band. Based on this estimate, find the amount of extra tax that would be raised if these properties were registered in the correct Valuation Band.

Question 8 **(50 marks)**

(a) A wind turbine, used to generate electricity, has three equally spaced blades 65 metres long.

  (i) Write down the size of the angle between two blades.

  (ii) Find the area of the disc traced out by one full rotation of the blades, correct to the nearest whole number.

  (iii) Find the area of the triangle formed by joining the tips of the three blades, correct to the nearest whole number.

(iv) The expected lifetime of the turbine is 25 years. On average, the turbine operates 31% of the time. The blades rotate 15 times per minute when the turbine is operating. Find the number of times the blades will rotate during the expected lifetime of the turbine (ignore leap years). Write your answer in the form $a \times 10^n$, where $1 \le a < 10$ and $n \in \mathbb{Z}$.

(b) Gráinne stood at a point B, which is on level ground 100 metres from the base of the tower supporting the blades, as shown. From there, she measured the angle of elevation to the top of the tower as 60°. Find the height of the tower, using Gráinne's measurements. Give your answer correct to the nearest metre.

60°

100 m $\quad B$

(c) Gráinne recognises that her measurement of the angle may not be totally accurate. She read elsewhere that the actual height of the tower is 154 m.

(i) If Gráinne measured the 100 m accurately, find the actual size of the angle at *B*, correct to the nearest degree.
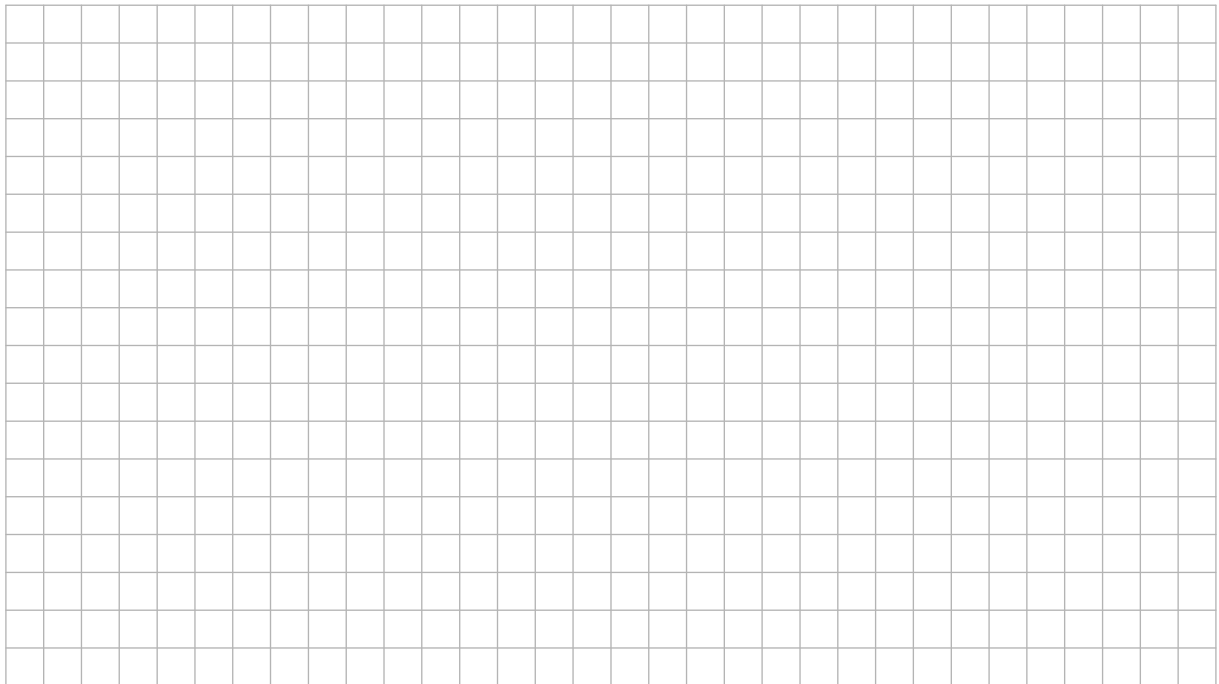
(ii) Find the percentage error in Grainne's measurement of the angle of elevation, correct to one decimal place.
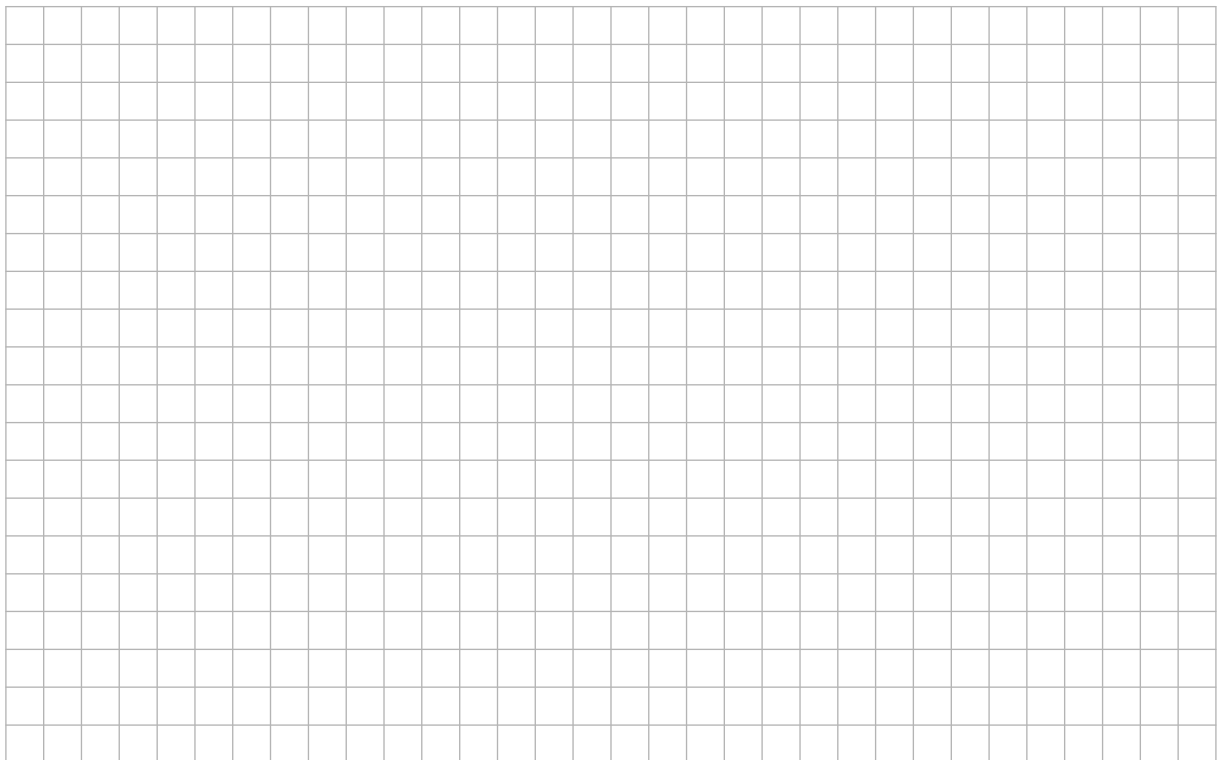
Question 9 (30 marks)

At an activity centre a zip-line, [BD], runs between two vertical poles, [AB] and [CD], on level ground, as shown. The point E is on the ground, directly below the zip-line.
$|AE| = 12$ m, $|BE| = 14$ m, $|CD| = 1 \cdot 95$ m, and $|EC| = 10$ m.
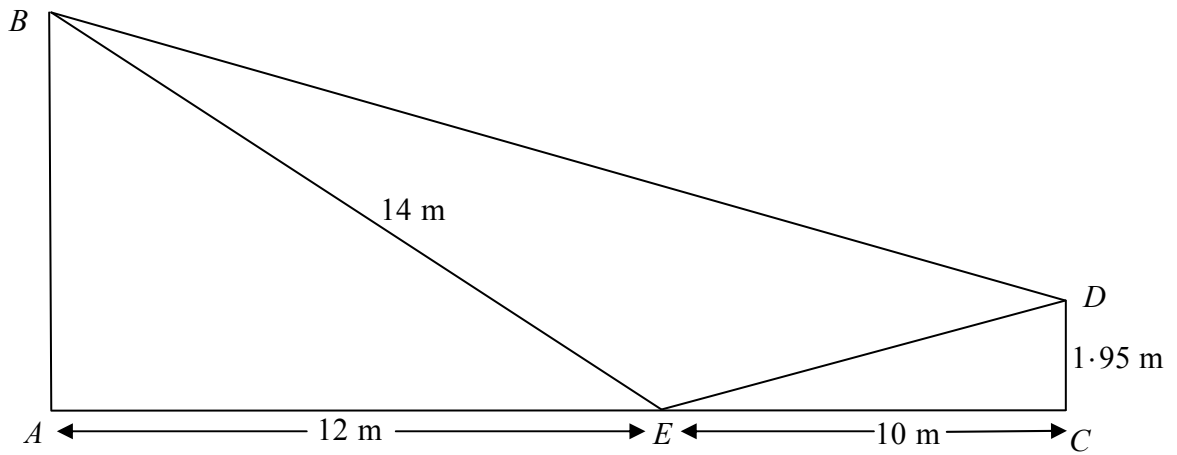
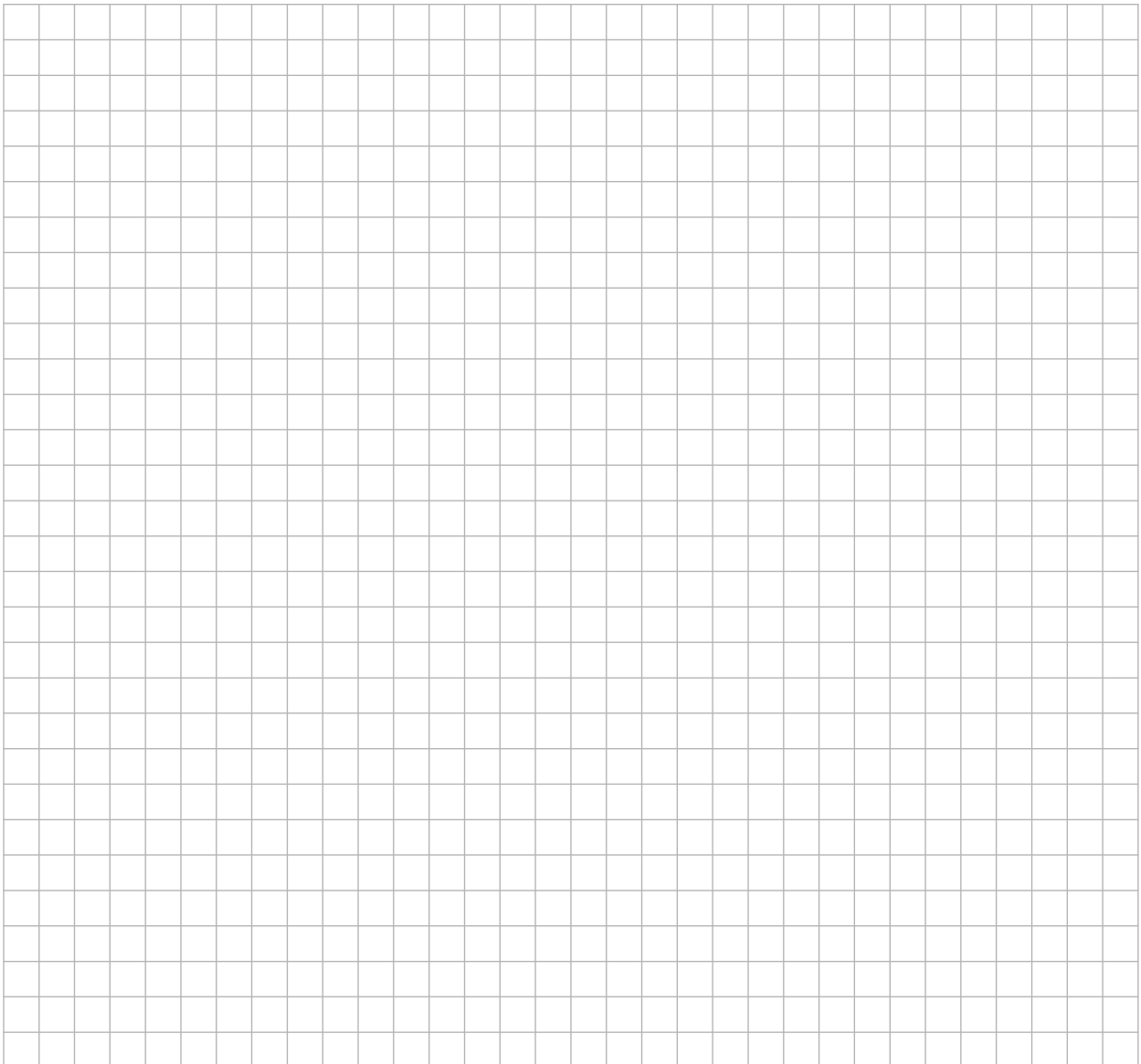**(a)** **(i)** Find the distance |ED|, correct to one decimal place.
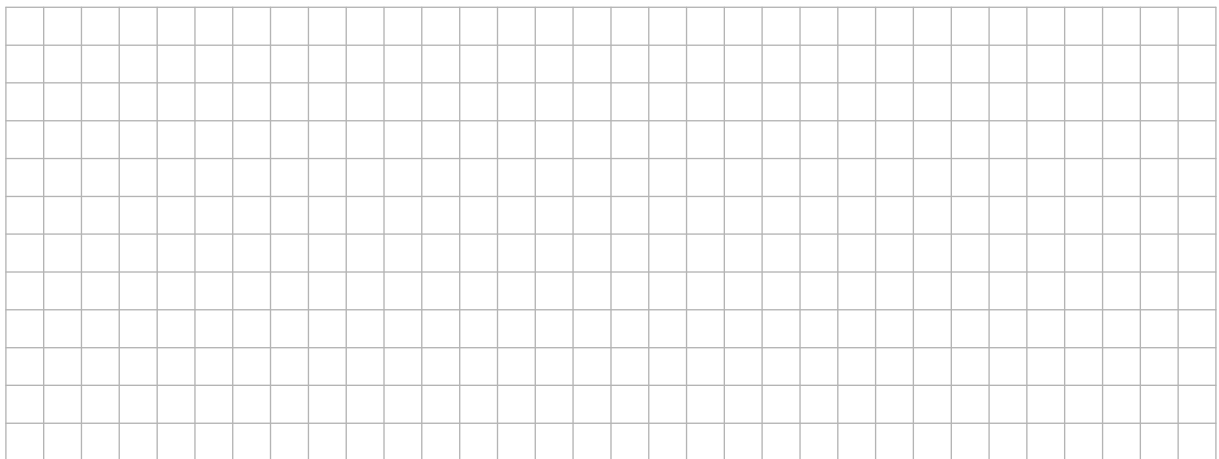
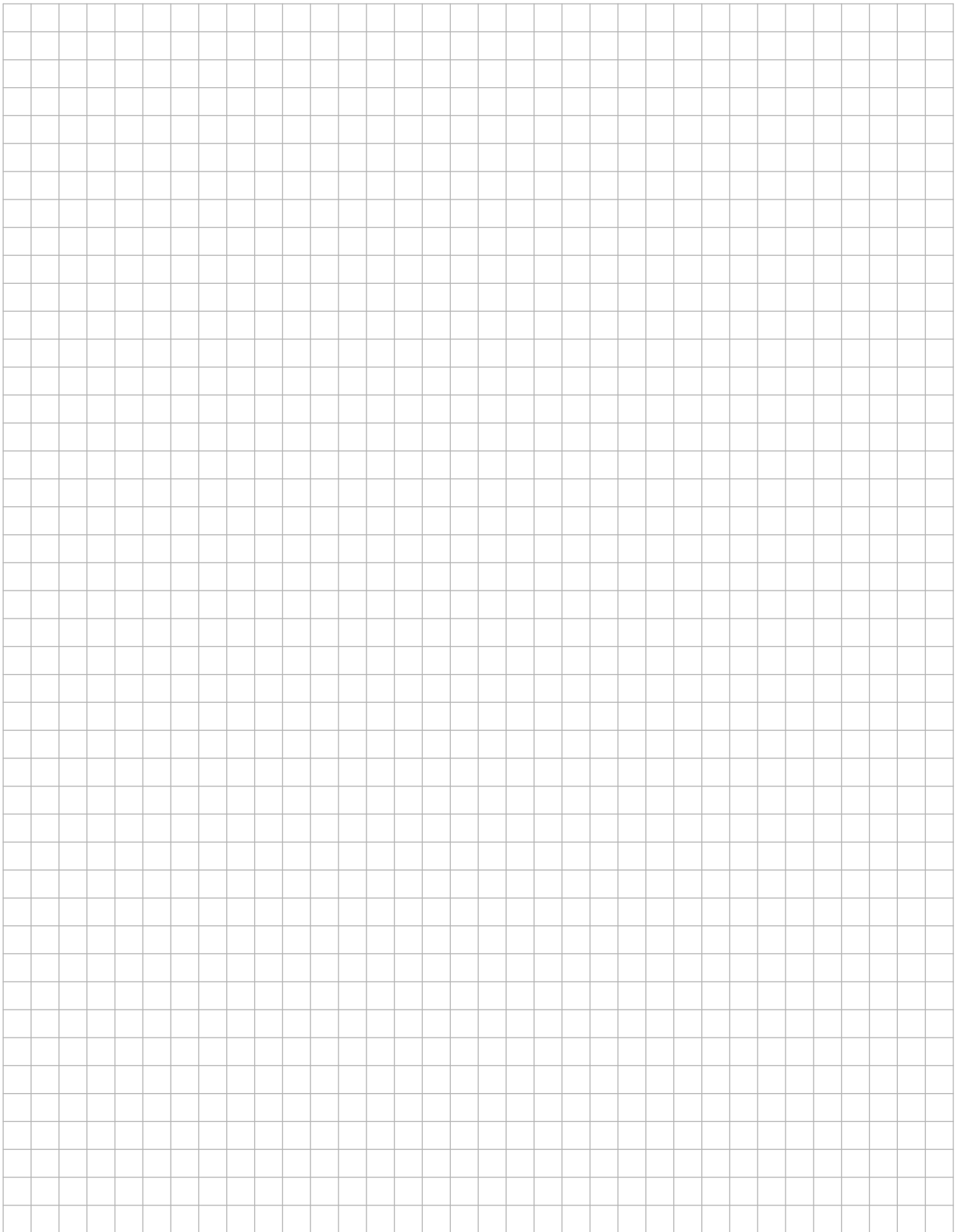**(ii)** Find $|\angle AEB|$, correct to the nearest degree.
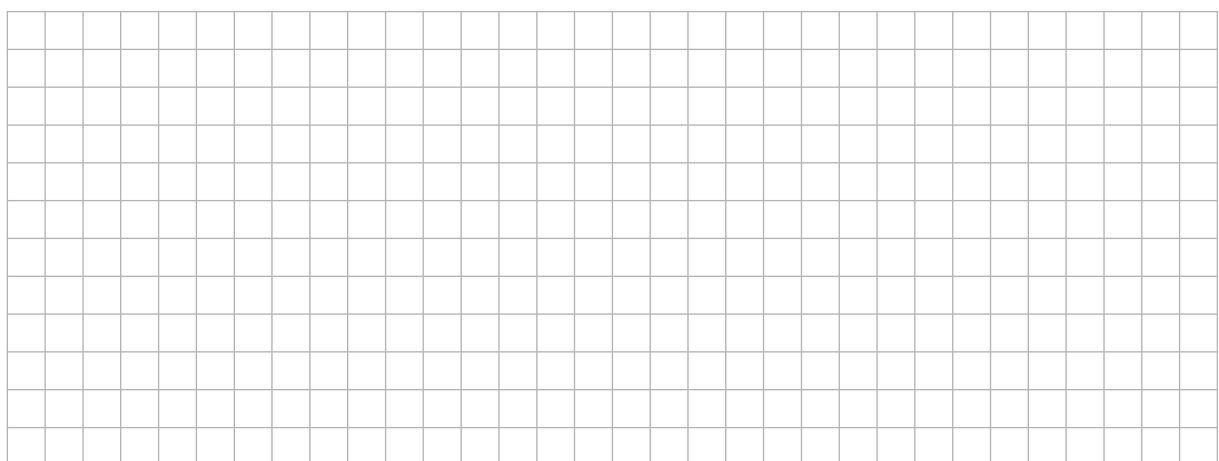
(b) **(i)** Find $|\angle DEB|$, given that $|\angle CED| = 11°$, correct to the nearest degree.
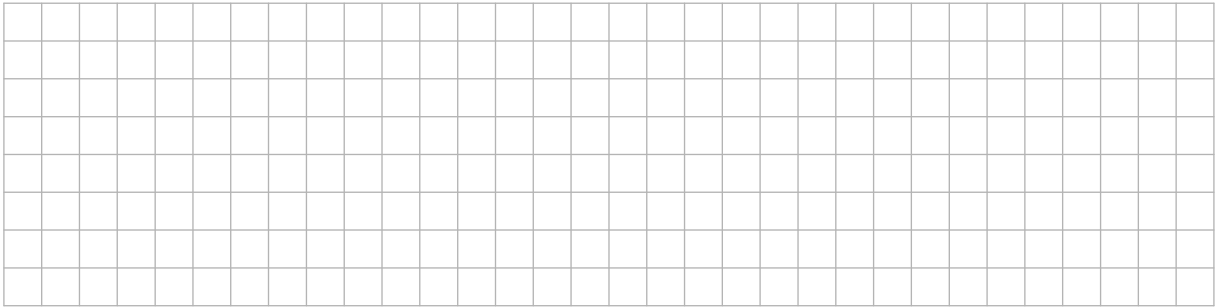
(ii) Hence, or otherwise, find the distance $|DB|$.
Give your answer correct to one decimal place.

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