

Algebra

Factors & Roots

The Factor Theorem

P1 - Algebra

If $(x - 3)$ and $(x + 2)$ are factors of the equation $f(x) = x^3 - 6x^2 + ax + b$, find the values of a and b .

Solution

Applying the Factor Theorem, $(x - 3)$ is a factor so $f(3) = 0$

$$\Rightarrow f(3) = (3)^3 - 6(3)^2 + a(3) + b = 0$$

$$\Rightarrow 27 - 54 + a(3) + b = 0$$

$$\Rightarrow 3a + b = 27 \quad \dots (i)$$

Similarly, $f(-2) = 0$

$$\Rightarrow f(-2) = (-2)^3 - 6(-2)^2 + a(-2) + b$$

$$\Rightarrow -8 - 24 - 2a + b$$

$$\Rightarrow -2a + b = 32 \quad \dots (ii)$$

Subtract (i) from (ii)

$$-2a + b = 32$$

$$\underline{3a + b = 27}$$

$$-5a = 5$$

$$\boxed{a = -1} \text{ and } \boxed{b = 30}$$

So the equation becomes $f(x) = x^3 - 6x^2 - x + 30$