Algebra

Algebraic Identities

P4 – Algebra

If $(x - t)^2$ is a factor of $x^3 + 3px + c$ for all values of x, find the relationship between t and p and the relationship between t and c.

Solution

$x^3 + 3px + c$ = (missing factor) $(x - t)^2$

To find the missing factor, we use long division:

$$
\begin{array}{r}
x + 2t \\
x^2 - 2xt + t^2 \overline{)\,x^3 \qquad + 3px + c} \\
\underline{x^3 - 2x^2t + t^2x} \\
2x^2t - t^2x + 3px + c \\
\underline{2x^2t - 4t^2x + 2t^3} \\
3t^2x - 2t^3 + 3px + c
\end{array}
$$

….. the remainder of the long division

Since the equation is true for all values of x, and there should be no remainder, we can say:

$3t^2x + 3px = 0$, therefore $t^2 + p = 0$ or $p = -t^2$

And $-2t^3 + c = 0$, therefore $c = 2t^3$