Algebra

Inequalities

Rational Inequalities

A ***rational inequality*** is an inequality whose numerator and denominator are both algebraic expressions.

P6 - Algebra

Solve the inequality $\dfrac{2x+4}{x+1} < 3,$ where x ∈ R and x ≠ −1

Solution

Recall that if we multiply both sides of an inequality by a negative number we will need to switch the direction of the inequality. However, since we don't know the value of x we don't know if the denominator is positive or negative and so we won't know if we need to switch the direction of the inequality or not.

So, we need to leave the rational expression in the inequality. Instead of multiplying across by (x + 1), we multiply by (x + 1)² which we know is positive.

So, we now get $\qquad \dfrac{(x+1)^2(2x+4)}{x+1} < 3(x+1)^2$

$\Rightarrow \qquad (x + 1)(2x + 4) < 3(x+1)^2$

$\Rightarrow \qquad 2x^2 + 6x + 4 < 3(x^2 + 2x + 1)$

$\Rightarrow \qquad 2x^2 + 6x + 4 < 3x^2 + 6x + 3$

$\Rightarrow \qquad -x^2 + 1 < 0$

$\Rightarrow \qquad x^2 - 1 > 0$

Now let $x^2 - 1 = 0$ and solve: $\qquad x^2 - 1 = 0$

$\Rightarrow \qquad x^2 = 1$

$\Rightarrow \qquad x = \pm\, 1$

$\therefore \qquad -1 > x > 1$