Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2016

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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State Examinations Commission

Leaving Certificate 2016

Model Solutions and Marking Scheme

Mathematics

Higher Level

Paper 1

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 4, 5	0, 2, 3, 4, 5	
10 mark scales	0, 10	0, 5, 10	0, 3, 7, 10	0, 2, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded. Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	5B
(b)	10C
(c)	10C

Question 2

(a)	10C
(b)	15C

Question 3

(a)(i)	5C
(ii)	5C
(iii)	5B
(b)	10C

Question 4

(a)	15D
(b)(i)	5C
(ii)	5D

Question 5

(a)(i)	10D
(ii)	5B
(b)(i)	5B
(ii)	5B

Question 6

(a)	10D
(b)(i)+(ii)	15D

Section B

Question 7

(a)(i)	10C
(a)(ii)	10C
(b)(i)	10C
(b)(ii)	10C

Question 8

(a)(i)	10C
(a)(ii)	5B
(a)(iii)	5B
(a)(iv)	10D
(b)(i)	10D
(b)(ii)	5B
(b)(iii)	10C

Question 9

(a)(i)	10C
(a)(ii)	10C
(a)(iii)	15D
(b)(i)	5B
(b) (ii)	10C
(b)(iii)	5B

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$-4-3i$	Scale 5B (0, 2, 5) *Partial Credit:* • real or imaginary part correct
(b)	$r=\sqrt{1^2+1^2}=\sqrt{2} \qquad \theta=\frac{\pi}{4}$ $(1+i)^8=\left\{\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right\}^8$ $(1+i)^8=\{16(\cos 2\pi+i\sin 2\pi)\}$ $(1+i)^8$= 16(1) = 16	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • correct answer without use of De Moivre's • modulus **or** argument correct • formula • statement of De Moivre's *High Partial Credit:* • $16(\cos 2\pi+i\sin 2\pi)$ **Note:** *not De Moivre and incorrect answer merits 0 marks*
(c)	$z=\frac{(2-i)\pm\sqrt{(-2+i)^2-4(3-i)}}{2}$ $=\frac{(2-i)\pm\sqrt{4-4i-1-12+4i}}{2}$ $=\frac{2-i\pm\sqrt{-9}}{2}$ $=\frac{2-i\pm 3i}{2}$ $=1-2i$ **or** $1+i$ **Or** $ax^2+bx+c=0$ $x^2-\left(\frac{-b}{a}\right)x+\frac{c}{a}=0$ Sum of roots $=-\frac{b}{a}$ $1+i+z_1=2-i$ $z_1=1-2i$	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • root formula with some substitution *High Partial Credit* • formula fully substituted **Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • equation rearranged • $-\frac{b}{a}$ *High Partial Credit* • correct substitution

Or

$(z-1-i)(z-z_1)$

$$= z^2 - z - zi - z.z_1 + z_1 + z_1 i$$
$$= z^2 - (1+i+z_1)z + z_1(1+i)$$
$$= z^2 + (-2+i)z + (3-i)$$
$$\Rightarrow z_1(1+i) = 3-i$$
$$z_1 = \frac{3-i}{1+i}.\frac{1-i}{1-i} = 1-2i$$

Or

Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- correct factor(s)

High Partial Credit
- identification of equal terms

Or

Or

$$\begin{array}{r}
z-1+2i \\
z-1-i\,\big)\,\overline{z^2-2z+iz+3-i} \\
\underline{z^2-z-iz} \qquad\qquad \\
-z+2iz+3-i \\
\underline{-z+1 \qquad +\mathrm{i}} \\
2iz+2-2i \\
\underline{2iz+2-2i}
\end{array}$$

$z-1+2i=0$

$z=1-2i$

Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- long division formulated correctly

High Partial Credit
- two correct lines in division

Or

Or

$$(1+i)(m+ni) = 3-i$$
$$(m-n)+(m+n)i = 3+(-1)i$$
$$m-n=3 \quad and \quad m+n=-1$$

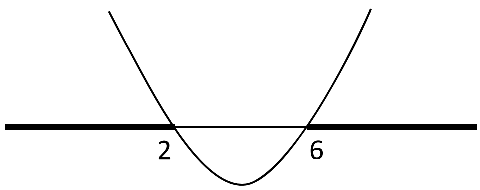
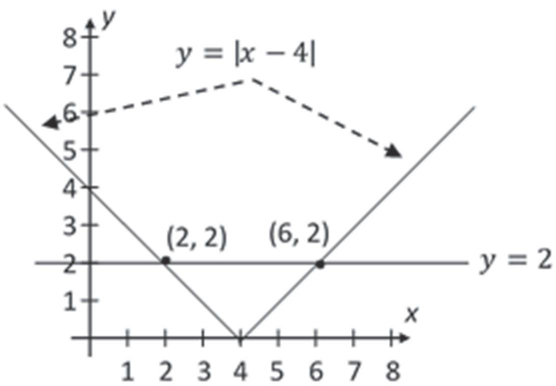
Solving $\quad m=1 \quad and \quad n=-2$

Scale 10C (0, 3, 7, 10)

Low Partial Credit:
- correct multiplication
- substitution of $(m+ni)$ into quadratic and stops

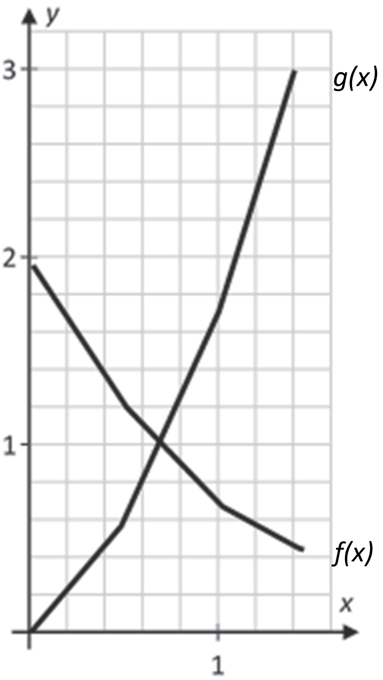
High Partial Credit
- identification of equal terms

Note: *substitution of* $(1+i)$ *merits 0 marks*

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$x^2 - 8x + 16 \geq 4$ $x^2 - 8x + 12 \geq 0$ $(x-2)(x-6) \geq 0$ $x = 2 \quad x = 6$ $\{x \mid x \leq 2\} \cup \{x \mid x \geq 6\}$ **Or** $x - 4 \geq 2 \ \cup x - 4 \leq -2$ $x \geq 6 \ \cup x \leq 2$ **Or** Graphical method (must indicate range on X-axis somehow) 	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • either side squared • one correct linear inequality written • stating range of natural numbers only *High Partial Credit:* • correct solutions to quadratic *Full Credit:* • correct answer without work **Note:** *use of natural numbers in range merits High Partial Credit at most*
	Or  $x \leq 2 \cup x \geq 6$	**Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • any one straight line *High Partial Credit:* • three straight lines

(b)		
	$$x = \frac{-3y-1}{2}$$ $$\left(\frac{-3y-1}{2}\right)^2 + \left(\frac{-3y-1}{2}\right)(y) + 2y^2 = 4$$ $$11y^2 + 4y - 15 = 0$$ $$(11y+15)(y-1) = 0$$ $$y = \frac{-15}{11} \ \ or \ \ y = 1$$ $$x = \frac{-3\left(\frac{-15}{11}\right)-1}{2} \quad or\ x = \frac{-3(1)-1}{2}$$ $$x = \frac{17}{11} \quad or\ x = -2$$ **or** $$y = \frac{-2x-1}{3}$$ $$x^2 + x\left(\frac{-2x-1}{3}\right) + 2\left(\frac{-2x-1}{3}\right)^2 = 4$$ $$11x^2 + 5x - 34 = 0$$ $$(11x-17)(x+2) = 0$$ $$x = \frac{17}{11} \quad \text{or } x = -2$$ $$y = \frac{-15}{11} \ \ \text{or} \ \ y = 1$$	Scale 15C (0, 5, 10,15) *Low Partial Credit:* • effort to isolate x (or y) *High Partial Credit:* • fully correct substitution into quadratic

<table>
<tr><th>Q3</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td>
<table>
<tr><td>x</td><td>0</td><td>0·5</td><td>1</td><td>$\ln(4)$</td></tr>
<tr><td>$f(x)=\frac{2}{e^x}$</td><td>2</td><td>1·21</td><td>0·74</td><td>0·5</td></tr>
<tr><td>$g(x)=e^x-1$</td><td>0</td><td>0·65</td><td>1·72</td><td>3</td></tr>
</table>
</td><td>Scale 5C (0, 2, 4, 5)
Low Partial Credit
• one entry correct

High Partial Credit
• 5 entries correct</td></tr>
<tr><td>(ii)</td><td>
</td><td>Scale 5C (0, 2, 4, 5)
Low Partial Credit
• one plot correct

High Partial Credit
• 5 plots correct
• one correct graph
• no labelling

Notes:
- straight lines <u>NOT</u> acceptable
- one clear label merits full credit
- one ambiguous label merits High Partial Credit at most</td></tr>
<tr><td>(iii)</td><td>$f(x)=g(x)$ when $x\approx 0\cdot 7$</td><td>Scale 5B (0, 2, 5)
Partial Credit
• point of intersection clearly indicated on graph, but value of x not stated</td></tr>
</table>

Q3	Model Solution – Continued	Marking Notes
(b)	$\dfrac{e^x - 1}{1} = \dfrac{2}{e^x}$ $e^{2x} - e^x = 2$ $(e^x)^2 - e^x - 2 = 0$ $(e^x - 2)(e^x + 1) = 0$ $e^x = 2$ **or** $e^x = -1$ $x = \ln 2$ or $x = 0{\cdot}693$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • substitution correct *High Partial Credit* • correct factors of quadratic • root formula correctly substituted $e^x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ **Note:** *oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most*
	Or $(e^x)^2 - e^x - 2 = 0$ Let $y = e^x \Rightarrow y^2 - y - 2 = 0$ $y = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ $= \dfrac{1 \pm \sqrt{1+8}}{2}$ $= \dfrac{1 \pm 3}{2}$ $\Rightarrow y = 2$ or $y = -1$ (not possible) $y = e^x \Rightarrow e^x = 2$ $x = \ln 2$ or $x = 0{\cdot}693$	**Or** Scale 10C (0, 3, 7, 10) *Low Partial Credit* • substitution correct *High Partial Credit* • root formula correctly substituted $y = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$ **Note:** *oversimplification of equation (i.e. not treating as quadratic) merits Low Partial Credit at most*

Q4	Model Solution – 25 Marks	Marking Notes
(a)	P_1: $8^1 - 1 = 7$ (divisible by 7) P_k: Assume $8^k - 1$ is divisible by 7 $8^k - 1 = 7M$ $8^k = 7M + 1$ P_{k+1}: $8^{k+1} - 1 = 8(8^k) - 1$ $= 8(7M + 1) - 1$ $= 56M + 7$ $= 7(8M + 1)$ P_{k+1} is divisible by 7 P_1 is true P_k true $\implies$ P_{k+1} is true So, P_{k+1} true whenever P_k true. Since P_1 true, then, by induction, P_n is true for all natural numbers ≥ 1 **Or** $P_{k+1} = 8^{k+1} - 1$ $= 8.8^k - 1$ $= (7 + 1).8^k - 1$ $= 7(8^k) + (8^k - 1)$ Obviously divisible by 7 From P_k So, P_{k+1} true whenever P_k true. Since P_1 true, then, by induction, P_n is true for all natural numbers ≥ 1	Scale 15D (0, 4, 7, 11, 15) *Low Partial Credit* • P_1 step *Mid Partial Credit* • P_k step • P_{k+1} step *High Partial Credit* • use of P_k step to prove P_{k+1} step **Note:** *accept* P_1 step, P_k step *and* P_{k+1} step *in any order*

(b) **(i)**	$p = \log_a 2$, $q = \log_a 3$ $\log_a \frac{8}{3} = \log_a 8 - \log_a 3$ $= \log_a (2)^3 - \log_a 3$ $= 3\log_a 2 - \log_a 3$ $= 3p - q$	Scale 5C (0, 2, 4, 5) *low Partial Credit* • $\log_a 8 - \log_a 3$ *High Partial Credit* • $\log_a 8 = 3\log_a 2$ (and/or $= 3p$)
(ii)	$\log_a \frac{9a^2}{16} = \log_a (3a)^2 - \log_a (2)^4$ $= 2\log_a 3 + 2\log_a a - 4\log_a 2$ $= 2q + 2(1) - 4p$ $= 2q + 2 - 4p$	Scale 5D (0, 2, 3, 4, 5) *Low Partial Credit* • $\log_a 9a^2 - \log_a 16$ *Mid Partial Credit* • $2\log_a 3$ • $2\log_a a$ • $4\log_a 2$ • $4p$ or $2q$ or 2 *High Partial Credit* • $2(\log_a 3 + \log_a a) - 4\log_a 2$ or equivalent

Q5	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	$(5x-9)^2 = (x-1)^2 + (4x)^2$ $8x^2 - 88x + 80 = 0$ $x^2 - 11x + 10 = 0$ $(x-1)(x-10) = 0$ $x = 1 \text{ or } x = 10$ $x = 10$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • any use of Pythagoras *Mid Partial Credit* • fully correct substitution *High Partial Credit* • both roots correct
(a) **(ii)**	Sides=9, 40, 41 $9^2 + 40^2 = 41^2$ $81 + 1600 = 1681$ $1681 = 1681$	Scale 5B (0, 2, 5) *Partial Credit* • 9 or 40 or 41 • using 1 or -10 from candidates work
(b) **(i)**	Function is bijective if inverse exists $f^{-1}(x) = \frac{x+2}{3}$ $\Rightarrow$ Function is injective. **or** Horizontal line test. **or** $f(a) = f(b)$ $3a - 2 = 3b - 2$ $\Longrightarrow a = b$ **or** $\forall\ a, b\ \varepsilon\ A, f(a) = f(b) \Rightarrow a = b$	Scale 5B (0, 2, 5) *Partial Credit* • $f^{-1}(x)$ written • $f(x)$ drawn • $f(a) = f(b)$
(b) **(ii)**	$f(x) = 3x - 2$ $f^{-1}(x) = \frac{x+2}{3}$	Scale 5B (0, 2, 5) *Partial Credit* • any relevant transpose

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$f(x+h)-f(x)=(2x+2h+4)^2-(2x+4)^2$ $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}=$ $\lim\limits_{h\to 0}\dfrac{(2x+2h+4)^2-(2x+4)^2}{h}$ $=\lim\limits_{h\to 0}\left(\dfrac{\begin{bmatrix}(4x^2+8hx+4h^2+16x+16h+16)\\-(4x^2+16x+16)\end{bmatrix}}{h}\right)$ $=\lim\limits_{h\to 0}\dfrac{8hx+4h^2+16h}{h}$ $=8x+16$ **or** $f(x)=(2x+4)^2=4x^2+16x+16$ $f(x+h)=4(x+h)^2+16(x+h)+16$ $=4x^2+8hx+4h^2+16x+16h+16$ $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $\lim\limits_{h\to 0}\dfrac{8hx+4h^2+16h}{h}$ $=8x+16$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • any $f(x+h)$ *Mid Partial Credit* • limit of $\dfrac{f(x+h)-f(x)}{h}$ *High Partial Credit* • limit of $\dfrac{(2x+2h+4)^2-(2x+4)^2}{h}$ **Notes:** - *omission of limit sign penalised once only* - *answer not from 1st Principles merits 0 marks*

(b) (i)+ (ii)	$$y = x.\sin\frac{1}{x}$$ $$\frac{dy}{dx} = \sin\frac{1}{x} + x\left(\cos\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$$ $$\frac{dy}{dx} = \sin\frac{1}{x} - \frac{1}{x}\cos\frac{1}{x}$$ $$\frac{dy}{dx} = \sin\frac{\pi}{4} - \frac{\pi}{4}\cos\frac{\pi}{4}$$ $$= 0{\cdot}15$$	Scale 15D (0, 4, 7, 11, 15) *Low Partial Credit* • any correct differentiation *Mid Partial Credit* • product rule applied *High Partial Credit* • correct differentiation **Note:** *one penalty for calculator in wrong mode*

Q7	Model Solution – 40 Marks	Marking Notes
(a) **(i)**	$v = \frac{4}{3}\pi r^3 \Rightarrow \frac{dv}{dr} = 4\pi r^2$ $\frac{dv}{dt} = 250 \text{ cm}^3/\text{s}$ $\frac{dr}{dt} = \frac{dr}{dv}.\frac{dv}{dt} = \frac{1}{4\pi r^2}.250$ $\frac{dr}{dt} = \frac{250}{4\pi 400} = \frac{5}{32\pi} \ \text{cm/s}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • work towards $\frac{dv}{dr}$ or $\frac{dv}{dt}$ or $\frac{dr}{dt}$ *High Partial Credit* • correct expression for $\frac{dr}{dt}$
(ii)	$a = 4\pi r^2 \Rightarrow \frac{da}{dr} = 8\pi r$ $\frac{da}{dt} = \frac{da}{dr}.\frac{dr}{dt} = 8\pi r.\frac{5}{32\pi}$ $= \frac{5(20)}{4}$ $= 25 \text{ cm}^2/\text{s}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • work towards $\frac{da}{dr}$ or $\frac{da}{dt}$ *High Partial Credit* • correct expression for $\frac{da}{dt}$
(b) **(i)**	$-x^2 + 10x = 0$ $x(-x + 10) = 0$ $x = 0 \quad \text{or} \quad x = 10$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • quadratic equation formed • gets $x = 0$ only *High Partial Credit* • quadratic factorised **Note:** $f'(x) = 0 \Rightarrow 2x - 10 = 0 \Rightarrow x = 5$ *merits 0 marks*
(ii)	$\frac{1}{10-0}\int_0^{10} (-x^2 + 10x)dx$ $= \frac{1}{10}\left[\frac{-x^3}{3} + 5x^2\right]_0^{10}$ $= \frac{1}{10}\left[\left(\frac{-1000}{3} + 500\right) - 0\right]$ $= \frac{-100}{3} + 50 = \frac{50}{3} \text{m}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • integration set up *High Partial Credit* • correct integration with some substitution

Q8	Model Solution – 55 Marks	Marking Notes
(a) **(i)**	$f(x) = -0{\cdot}274x^2 + 1{\cdot}193x + 3{\cdot}23$ $f'(x) = -0{\cdot}548x + 1{\cdot}193 = 0$ $x = 2{\cdot}177 \text{ m}$ $f(2{\cdot}177) = -0{\cdot}274(2{\cdot}177)^2 + 1{\cdot}193(2{\cdot}177) + 3{\cdot}23$ $= -1{\cdot}2986 + 2{\cdot}5972 + 3{\cdot}23$ $= 4{\cdot}529 \text{ m}$ *or* $-0{\cdot}274(x^2 - \frac{1193}{274}x - \frac{1615}{137})$ $-0{\cdot}274(x - \frac{1193}{548})^2 + 4.5285$ Max Height = 4·529 m	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • any correct differentiation • effort made at completing square • trial and error with more than one value of x tested *High Partial Credit* • x value correct **Note:** *if correct answer by trial and error, must show points on each side of max point to be lower to earn full credit*
(ii)	$\tan\theta = -0{\cdot}548(4{\cdot}5) + 1{\cdot}193$ $\tan\theta = -1{\cdot}273$ $\theta = 51{\cdot}8° = 52°$	Scale 5B (0, 2, 5) *Partial Credit* • tan **Note:** *right angled triangles may appear in diagram given in equation*
(iii)	Map $A \to C$ $(-0{\cdot}5, 2{\cdot}565) \to (0, 2)$ $2{\cdot}177 - (-0{\cdot}5) = 2{\cdot}677$ $4{\cdot}529 - 0{\cdot}565 = 3{\cdot}964$ $(2{\cdot}177, 4{\cdot}529) \to (2{\cdot}677, 3{\cdot}964)$	Scale 5B (0, 2, 5) *Partial Credit* • $(-0{\cdot}5, 2{\cdot}565) \to (0, 2)$

(iv)	$g(x) = ax^2 + bx + c$ $C(0,2) \in g(x) \Rightarrow c = 2$ $B(4{\cdot}5, 3{\cdot}05) \in g(x)$ $3{\cdot}05 = a(4{\cdot}5)^2 + b(4{\cdot}5) + 2$ $\Longrightarrow 20{\cdot}25a + 4{\cdot}5b = 1{\cdot}05$ ... **(i)** $g'(x) = 2ax + b = 0$ $\Longrightarrow 2a(2{\cdot}677) + b = 0$ $5{\cdot}354a + b = 0$ ... **(ii)** From **(i)** and **(ii)** $a = -0{\cdot}273$ $b = 1{\cdot}462$ $g(x) = -0{\cdot}273x^2 + 1{\cdot}462x + 2$ *[Note: a third equation that could be used is* $3{\cdot}964 = a(2{\cdot}677)^2 + b(2{\cdot}677) + 2$ ... **(iii)**]	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • *c* value found • relevant equation in *a*, *b* and/or *c* *Mid Partial Credit* • formulated correctly any two equations *High Partial Credit* • formulated correctly any three equations **Note**: $ax^2 + bx + c$ *not in an equation merits 0 marks*
	Or	**Or**
	Equation of parabola with vertex (h, k): $g(x) = a(x-h)^2 + k$ $C(0,2)$ on curve: $(h, k) = (2{\cdot}677, 3.964)$ $2 = a(-2{\cdot}677)^2 + 3{\cdot}964$ $-1{\cdot}964 = a(7{\cdot}166329)$ $a = -0{\cdot}27405 = -0{\cdot}274$ Parabola: $g(x) = -0{\cdot}274[(x - 2{\cdot}677)^2] + 3{\cdot}964$ **or** $g(x) = f(x - 0{\cdot}5) - 0{\cdot}565$ $g(x) = -0{\cdot}274(x-0{\cdot}5)^2 + 1{\cdot}193(x - 0{\cdot}5) + 3{\cdot}23 - 0{\cdot}565$ $g(x) = -0{\cdot}274x^2 + 1{\cdot}467x + 2$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • equation of curve • use of C *Mid Partial Credit* • using peak value *High Partial Credit* • value of *a* found

(b) **(i)**	200 m Race: $$y = a(b - x)^c$$ $$y = 4{\cdot}99087(42{\cdot}5 - 23{\cdot}8)^{1{\cdot}81}$$ $$y = 1000$$ Javelin: $$y = a(x - b)^c$$ $$y = 15{\cdot}9803(58{\cdot}2 - 3{\cdot}8)^{1{\cdot}04}$$ $$y = 1020$$	Scale 10D (0, 2, 5, 8, 10) *Low Partial Credit* • some relevant substitution into one formula *Mid Partial Credit* • one value of y found • some relevant substitution into both formulas *High Partial Credit* • one value correct and some relevant substitution into second formula • uses incorrect formula (once only)
(ii)	$$y = a(x - b)^c$$ $$1295 = 15{\cdot}9803(x - 3{\cdot}8)^{1{\cdot}04}$$ $$81{\cdot}0373 = (x - 3{\cdot}8)^{1{\cdot}04} = z^{1{\cdot}04}$$ $$\log z = \frac{\log 81{\cdot}0373}{1{\cdot}04}$$ $$z = 68{\cdot}4343 = (x - 3{\cdot}8)$$ $$x = 72{\cdot}2343 = 72{\cdot}23 \text{ m}$$	Scale 5B (0, 2, 5) *Partial Credit* • some relevant substitution into formula
(iii)	$$y = a(b - x)^c$$ $$1087 = 0{\cdot}11193(254 - 121{\cdot}84)^c$$ $$\frac{1087}{0{\cdot}11193} = (132{\cdot}16)^c$$ $$\log 9711{\cdot}426 = c \log 132{\cdot}16$$ $$c = \frac{\log 9711{\cdot}426}{\log 132{\cdot}16} = 1{\cdot}88$$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • some relevant substitution into formula *High Partial Credit* • fully correct substitution into formula

Q9	Model Solution – 55 Marks	Marking Notes
(a)(i)	$4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$ $S_n = \frac{a(1-r^n)}{1-r}$ $S_n = \frac{4\left(1-\left(\frac{1}{2}\right)^n\right)}{1-\frac{1}{2}} = 7{\cdot}9375$ $-\frac{1}{2^n} = -\frac{1}{128}$ $n = 7$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • some listing of terms • S_n formula *High Partial Credit* • listing of exactly 7 correct terms • formula fully substituted
(a) **(ii)**	$S_\infty = \frac{a}{1-r}$ $S_\infty = \frac{4}{1-\frac{1}{2}} = 8$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • S_∞ formula *High Partial Credit* • formula fully substituted

	1	2	3	4	5	6	7	8	9
Chg x	+4	0	-1	0	$\frac{1}{4}$	0	$-\frac{1}{16}$	0	$\frac{1}{64}$
Chg y	0	2	0	$-\frac{1}{2}$	0	$\frac{1}{8}$	0	$-\frac{1}{32}$	0

(a) (iii)	$S_\infty = \frac{4}{1-\left(-\frac{1}{4}\right)} = 3{\cdot}2 = \frac{16}{5}$ $S_\infty = \frac{2}{1-\left(-\frac{1}{4}\right)} = 1{\cdot}6 = \frac{8}{5}$ $\left(\frac{16}{5}, \ \frac{8}{5}\right)$ or $(3{\cdot}2,\ 1{\cdot}6)$	Scale 15D (0, 4, 7, 11, 15) *Low Partial Credit* • 2 extra entries correct in either row *Mid Partial Credit* • either row fully correct *High Partial Credit* • one co-ordinate correct **Notes:** - *need to see S_∞ correctly used to move beyond Mid Partial Credit* - *no S_∞ merits Mid Partial Credit at most*
(b) (i)	G_5 =Female,Male,Female,Female,Male	Scale 5B (0, 2, 5) *Partial Credit* • one correct entry
(b) (ii)	$G_6 = G_5 + G_4 = 5 + 3 = 8$ $G_7 = G_6 + G_5 = 8 + 5 = 13$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • $G_6 = G_5 + G_4$ • $G_7 = G_6 + G_5$ • G_7 or G_6 correct • 8 and/or 13 without work *High Partial Credit* • correct substitution in both

(b) (iii)	$G_3 = \dfrac{(1+\sqrt{5})^3 - (1-\sqrt{5})^3}{2^3\sqrt{5}} = 2$ $(1+\sqrt{5})^3 = \left(1 + 3\sqrt{5} + 3\sqrt{5}^2 + \sqrt{5}^3\right)$ $= 16 + 8\sqrt{5}$ $(1-\sqrt{5})^3 = \left(1 - 3\sqrt{5} + 3\sqrt{5}^2 - \sqrt{5}^3\right)$ $= 16 - 8\sqrt{5}$ $G_3 = \dfrac{6\sqrt{5} + 2\sqrt{5}^3}{8\sqrt{5}}$ $= \dfrac{6 + 2\sqrt{5}^2}{8} = \dfrac{16}{8} = 2 \quad \text{Q.E.D.}$	Scale 5B (0, 2, 5) *Partial Credit* • some correct substitution • using approximate value for $\sqrt{5}$ • $G_3 = 2$ • some effort at cubing **Note:** *use of* $\sqrt{5}$ *as approximation, even if rounded off to* 2 *at end of work merits at most Partial Credit*

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Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2016

Model Solutions and Marking Scheme

Mathematics

Higher Level

Paper 2

Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales		0, 2, 5	0, 2, 4, 5		
10 mark scales		0, 5, 10	0, 3, 7, 10	0, 3, 5, 8, 10	
15 mark scales			0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales					
25 mark scales					

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response
- correct response

B-scales (three categories)
- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)
- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)
- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)
- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded. Thus, for example, in *scale 10C*, 9 marks may be awarded. Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)	10C
(b)	15D

Question 2

(a)	10C
(b)	15D

Question 3

(a)	15C
(b)	10D

Question 4

(a)(i)	15C
(a)(ii)	5C
(b)	5C

Question 5

(a) (i)	5B
(ii)	10C
(b)	10C

Question 6

(a)	10C
(b)	10C
(c)	5C

Section B

Question 7

(a)(i)	10C
(a)(ii)	10B
(a)(iii)	10C
(a)(iv)	10C
(a)(v)	10D
(b)	5C

Question 8

(a)	5C
(b)	5B
(c)	5C
(d)(i)	10C
(d)(ii)	10C
(e)	5B
(f)	5B

Question 9

(a)(i)	10D
(a)(ii)	5C
(a)(iii)	15D
(b)	10C
(c)	5B
(d)	5C

Model Solutions & Detailed Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her Advising Examiner.

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$\text{Slope } AC = -\frac{2}{3}$ $\text{perp. slope} = \frac{3}{2}$ $y - 3 = \frac{3}{2}(x - 5)$ $3x - 2y = 9$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • slope formula with some relevant substitution • 3 = 5*m*+*c* • $y - y_1 = m(x - x_1)$ with $x_1$or y_1 or both substituted *High Partial Credit* • perpendicular slope • equation of line through *B* parallel to *AC*
(b)	Point of intersection of the altitudes $\text{Slope } AB = \frac{3+2}{5-6} = -\frac{5}{1}$ $\text{perp. slope} = \frac{1}{5}$ $y - 4 = \frac{1}{5}(x + 3)$ $x - 5y + 23 = 0$ Orthocentre: $3x - 2y = 9 \cap x - 5y = -23$ $\Rightarrow y = 6 \quad x = 7$ $(7, 6)$ **or** If BC chosen: $\text{Slope } BC = \frac{3-4}{5+3} = -\frac{1}{8}$ $\text{perp. slope} = 8$ Equation of altitude: $y + 2 = 8(x - 6)$ Equation: $8x - y = 50$ Orthocentre: $3x - 2y = 9 \cap 8x - y = 50$ $\Rightarrow y = 6 \quad x = 7$ $(7, 6)$	Scale 15D (0, 4, 7,11,1 5) *Low Partial Credit* • demonstration of understanding of orthocentre (e.g. mentions altitude) • slope formula with some relevant substitution • altitude from part **(a)** *Mid Partial Credit* • equation of an altitude other than **(a)** • some relevant substitution towards finding a second altitude and altitude from **(a)** • correct construction *High Partial Credit* • two correct altitudes • correct construction with orthocentre (7, 6)

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$y-6=\frac{1}{7}(x+1)$ $x-7y+43=0$	Scale 10C (0, 3, 7, 10) *Low Partial Credit:* • equation of line formula with some relevant substitution *High Partial Credit:* • equation of line not in required form
(b)	$D=\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $D=\frac{\lvert 3(-g)+4(-f)-21\rvert}{\sqrt{3^2+4^2}}$ $25=\lvert-3g-4f-21\rvert$ $-3g-4f-21=\pm 25$ $\Longrightarrow 3g+4f=-46\ \ldots\ \text{(i)}$ and $3g+4f=4\ \ldots\ \text{(ii)}$ But $(-g,-f)\ \in x-7y+43=0$ $\Rightarrow -g+\ 7f+43=0\ \ldots\ \text{(iii)}$ $\Rightarrow g=\ 7f+43$ Solving : $g=\ 7f+43$ and $3g+4f=-46$ $f=-7$ and $g=-6$ Centre (6, 7) $(x-6)^2+(y-7)^2=25$ **or** Solving: $g=\ 7f+43$ and $3g+4f=4$ $f=-5$ and $g=8$ Centre (- 8, 5) $(x+8)^2+(y-5)^2=25$	Scale 15D (0, 4, 7,11,1 5) *Low Partial Credit* • some correct substitution into relevant formula (line, circle, perpendicular distance). *Mid Partial Credit* • one relevant equation in *g* and *f* • (either(i) or (ii) or (iii)) *High Partial Credit* • two relevant equations (either (i) and (iii) or (ii) and (iii))

<table>
<tr><th>Q3</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$$\frac{2\cos\frac{7A+A}{2}\cos\frac{7A-A}{2}}{2\cos\frac{7A+A}{2}\sin\frac{7A-A}{2}}$$

$$\frac{2\cos 4A\cos 3A}{2\cos 4A\sin 3A}$$

$$=\frac{\cos 3A}{\sin 3A}$$

$$=\cot 3A$$

</td><td>

Scale 15C (0, 5, 10, 15)

Low Partial Credit

- sum to product formula with some substitution

High Partial Credit

- sum to product formula fully substituted

</td></tr>
<tr><td>(b)</td><td>

Method 1:

$$\cos^2\theta=\frac{1}{2}(1+\cos 2\theta)$$

$$=\frac{1}{2}\left(1+\frac{1}{9}\right)=\frac{5}{9}$$

$$\cos\theta=\pm\frac{\sqrt{5}}{3}$$

or

Method 2:

$\cos 2\theta=1-2\sin\theta=\frac{1}{9}$

$$9-18\sin^2\theta=1$$

$\sin^2\theta=\frac{4}{9}\Longrightarrow\sin\theta=\pm\frac{2}{3}\Longrightarrow\cos\theta=\pm\frac{\sqrt{5}}{3}$

or

Method 3:

$$\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1}{9}$$

$$9-9\tan^2\theta=1+\tan^2\theta$$

$$\tan^2\theta=\frac{4}{5}$$

$$\Longrightarrow\tan\theta=\pm\frac{2}{\sqrt{5}}\Longrightarrow\cos\theta=\pm\frac{\sqrt{5}}{3}$$

</td><td>

Scale 10D (0, 3, 5, 8, 10)

Low Partial Credit

- Use of a relevant formula in $\cos 2\theta$
- $\cos^{-1}\left(\frac{1}{9}\right)=83{\cdot}62°$
- $\theta=41{\cdot}8°$

Mid Partial Credit

- correct substitution (method 1)
- expression in $\sin^2\theta$ (method 2)
- expression in $\tan^2\theta$ (method 3)
- expression in $\cos^2\theta$ (method 4)
- $\theta=41{\cdot}8°$ and $\theta=132{\cdot}2°$ or $\theta=221{\cdot}8°$

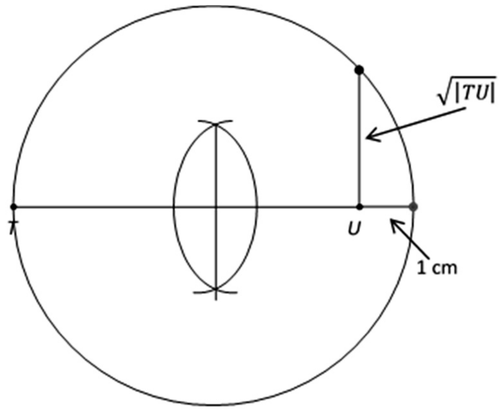
High Partial Credit

- one value only (e.g. $+\frac{\sqrt{5}}{3}$)
- values found for $\cos 41{\cdot}8°$ and $\cos 138{\cdot}2°$ or $\cos 221{\cdot}8°$

</td></tr>
</table>

	or Method 4: $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $1-\cos^2\theta = \frac{1}{2}(1-\cos 2\theta)$ $2-2\cos^2\theta = 1-\cos 2\theta$ $\cos^2\theta = \frac{1+\cos 2\theta}{2} = \frac{1+\frac{1}{9}}{2}$ $\cos^2\theta = \frac{5}{9}$ $\cos\theta = \pm\frac{\sqrt{5}}{3}$	

Q4	Model Solution – 25 Marks	Marking Notes
(a) **(i)**	$\lvert\angle ABD\rvert = \lvert\angle CBD\rvert = 90°$.......(i) $\lvert\angle BDC\rvert + \lvert\angle BCD\rvert = 90°$ ...*angles in triangle sum to* $180°$ $\lvert\angle ADB\rvert + \lvert\angle BDC\rvert = 90°$ .... *angle in semicircle* $\lvert\angle ADB\rvert + \lvert\angle BDC\rvert = \lvert\angle BDC\rvert + \lvert\angle BCD\rvert$ $\lvert\angle ADB\rvert = \lvert\angle BCD\rvert$........(ii) $\therefore$ *Triangles are equiangular* (*or similar*) **or** $\lvert\angle ABD\rvert = \lvert\angle CBD\rvert = 90°$.......(i) $\lvert\angle DAB\rvert = \lvert\angle DAC\rvert$ *same angle* $\Rightarrow \lvert\angle ADB\rvert = \lvert\angle DCA\rvert$ (*reasons as above*) *which is also* $\angle DCB$.........(ii)	Scale 15C (0, 5, 10, 15) *Low Partial Credit* • identifies one angle of same size in each triangle *High Partial Credit* • identifies second angle of same size in each triangle • implies triangles are similar without justifying (ii) in model solution or equivalent
(a) **(ii)**	$\frac{y}{1} = \frac{x}{y}$ $\Rightarrow y^2 = x$ $y = \sqrt{x}$ **or** $\lvert AD\rvert^2 + \lvert DC\rvert^2 = \lvert AC\rvert^2$ $\lvert AD\rvert = \sqrt{x^2 + y^2}$ $\lvert DC\rvert = \sqrt{y^2 + 1}$ $x^2 + y^2 + y^2 + 1 = (x + 1)^2$ $2y^2 = 2x$ $y = \sqrt{x}$ **Or** $\frac{\sqrt{x^2 + y^2}}{\sqrt{y^2 + 1}} = \frac{y}{1} \Rightarrow x^2 + y^2 = y^2(y^2 + 1)$ $y^4 = x^2 \Rightarrow y^2 = x \Rightarrow y = \sqrt{x}$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • one set of corresponding sides identified • indicates relevant use of Pythagoras *High Partial Credit* • corresponding sides fully substituted • expression in y^2 or y^4, i.e. fails to finish

(b)	Construction  	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • perpendicular line drawn at U or T • relevant use of 1 cm length • mid point of incorrect extended segment constructed *High Partial Credit* • correct mid-point constructed

<table>
<tr><th>Q5</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td><table><tr><td>John</td><td>✓</td><td>✓</td><td>×</td><td>✓</td></tr><tr><td>David</td><td>✓</td><td>×</td><td>✓</td><td>✓</td></tr><tr><td>Mike</td><td>×</td><td>✓</td><td>✓</td><td>✓</td></tr></table></td><td>Scale 5B (0, 2, 5)
Partial Credit
• 1 correct column</td></tr>
<tr><td>(a)
(ii)</td><td>$$P(win) = \left(\frac{1}{5}\times\frac{1}{6}\times\frac{3}{4}\right) + \left(\frac{1}{5}\times\frac{5}{6}\times\frac{1}{4}\right) + \left(\frac{4}{5}\times\frac{1}{6}\times\frac{1}{4}\right) + \left(\frac{1}{5}\times\frac{1}{6}\times\frac{1}{4}\right) = \frac{13}{120}$$</td><td>Scale 10C (0, 3, 7, 10)
Low Partial Credit
• one correct triple (numerical or descriptive)
• probability of any one Miss

High Partial Credit
• 4 correct triples (numerical)</td></tr>
<tr><td>(b)</td><td>$$P(A \cap B) = P(A) \times P(B)$$ $$0{\cdot}1 = (x + 0{\cdot}1) \times 0{\cdot}4$$ $$0{\cdot}4x = 0{\cdot}06$$ $$x = 0{\cdot}15$$
or
$$P(A|B) = P(A)$$ $$\frac{0{\cdot}1}{0{\cdot}4} = x + 0{\cdot}1$$ $$x = 0{\cdot}15$$</td><td>Scale 10C (0, 3, 7, 10)
Low Partial Credit
• formula written or implied
• writes P(A) = $x + 0{\cdot}1$

High Partial Credit
• formula fully substituted</td></tr>
</table>

Q6	Model Solution – 25 Marks	Marking Notes

(a)

$$P(M,3,3) = \frac{1}{26} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{2600}$$

Marking Notes

Scale 10C (0, 3, 7, 10)

Low Partial Credit

- any correct relevant probability

High Partial credit

- correct probabilities but not expressed as single fraction or equivalent

Note: Accept correct answer without supporting work

(b)

Event	Payout	Prob (P(x))	x.P(x)
Win	1000	$\frac{1}{2600}$	$\frac{1000}{2600}$
letter 1 No.	50	$\frac{9}{2600}$	$\frac{450}{2600}$
letter 2nd No	50	$\frac{9}{2600}$	$\frac{450}{2600}$
letter only	50	$\frac{81}{2600}$	$\frac{4050}{2600}$
Fail to win	0		0

$$\sum x.P(x) = \frac{5950}{2600} = 2{\cdot}29$$

Club loses 29 cent per play

Or

Event	Pay out	Prob (P(x))	x.P(x)
Win	-998	$^{1}/_{2600}$	$^{-998}/_{2600}$
letter + 1st No.	-48	$^{9}/_{2600}$	$^{-432}/_{2600}$
Letter + 2nd No	-48	$^{9}/_{2600}$	$^{-432}/_{2600}$
letter only	-48	$^{81}/_{2600}$	$^{-3888}/_{2600}$
Fail to Win	$+2$	$^{2500}/_{2600}$	$^{5000}/_{2600}$

$$\sum x.P(x) = -\frac{750}{2600} = -29 \text{ cent}$$

Marking Notes

Scale 10C (0, 3, 7, 10)

Low Partial Credit

- 1 correct entry to table

High Partial Credit

- all entries correct but fails to finish or finishes incorrectly
- no conclusion

(c)	Profit = Revenue – Pay-out $600 = 845(x - 2{\cdot}29)$ $$x = \frac{600 + 845(2{\cdot}29)}{845}$$ $$x = 3$$ **or** $$\frac{600}{845} = 0{\cdot}71$$ $$0{\cdot}71 + 2{\cdot}29 = 3$$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • links profit, revenue and payout *High partial Credit* • formula fully substituted

Q7	Model Solution – 55 Marks	Marking Notes
(a) (i)	$\|EC\|^2 = 3^2 + 2{\cdot}5^2 = 15{\cdot}25$ $\|EC\| = \sqrt{15{\cdot}25}$ $\|EC\| = 3{\cdot}905$ $\Rightarrow \|AC\| = 1{\cdot}9525$ $= 1{\cdot}95$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • Pythagoras with relevant substitution *High Partial Credit* • $\|EC\|$ correct • $\|AC\| = \frac{1}{2}\sqrt{15{\cdot}25}$
(a) (ii)	$\tan 50° = \frac{\|AB\|}{1{\cdot}95}$ $\|AB\| = 1{\cdot}95(1{\cdot}19175) = 2{\cdot}23239$ $\|AB\| = 2{\cdot}3$	Scale 10B (0, 5, 10) *Partial Credit* • tan formulated correctly
(a) (iii)	$\|BC\|^2 = 1{\cdot}95^2 + 2{\cdot}3^2$ $\|BC\| = 3 \cdot 015377$ $\|BC\| = 3$ Also: $\sin 40° = \frac{1{\cdot}95}{\|BC\|}$ **or** $\cos 40° = \frac{2{\cdot}3}{\|BC\|}$ **or** $\cos 50° = \frac{1{\cdot}95}{\|BC\|}$ **or** $\sin 50° = \frac{2{\cdot}3}{\|BC\|}$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • Pythagoras with relevant substitution *High Partial Credit* • Pythagoras fully substituted • $\|BC\| = \frac{1{\cdot}95}{\sin 40°}$ (i.e. $\|BC\|$isolated)
(a) (iv)	$3^2 = 3^2 + 2{\cdot}5^2 - 2(3)(2{\cdot}5)\cos\propto$ $15\cos\propto = 6{\cdot}25$ $\propto = 65°$ **or** $\cos\propto = \frac{1{\cdot}25}{3}$ $\propto = 65°$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • cosine rule with some relevant substitution • cosine ratio with some relevant substitutions • identifies three sides of triangle *BCD* *High Partial Credit* • cosine rule with full relevant substitutions • cosine ratio with full relevant substitutions

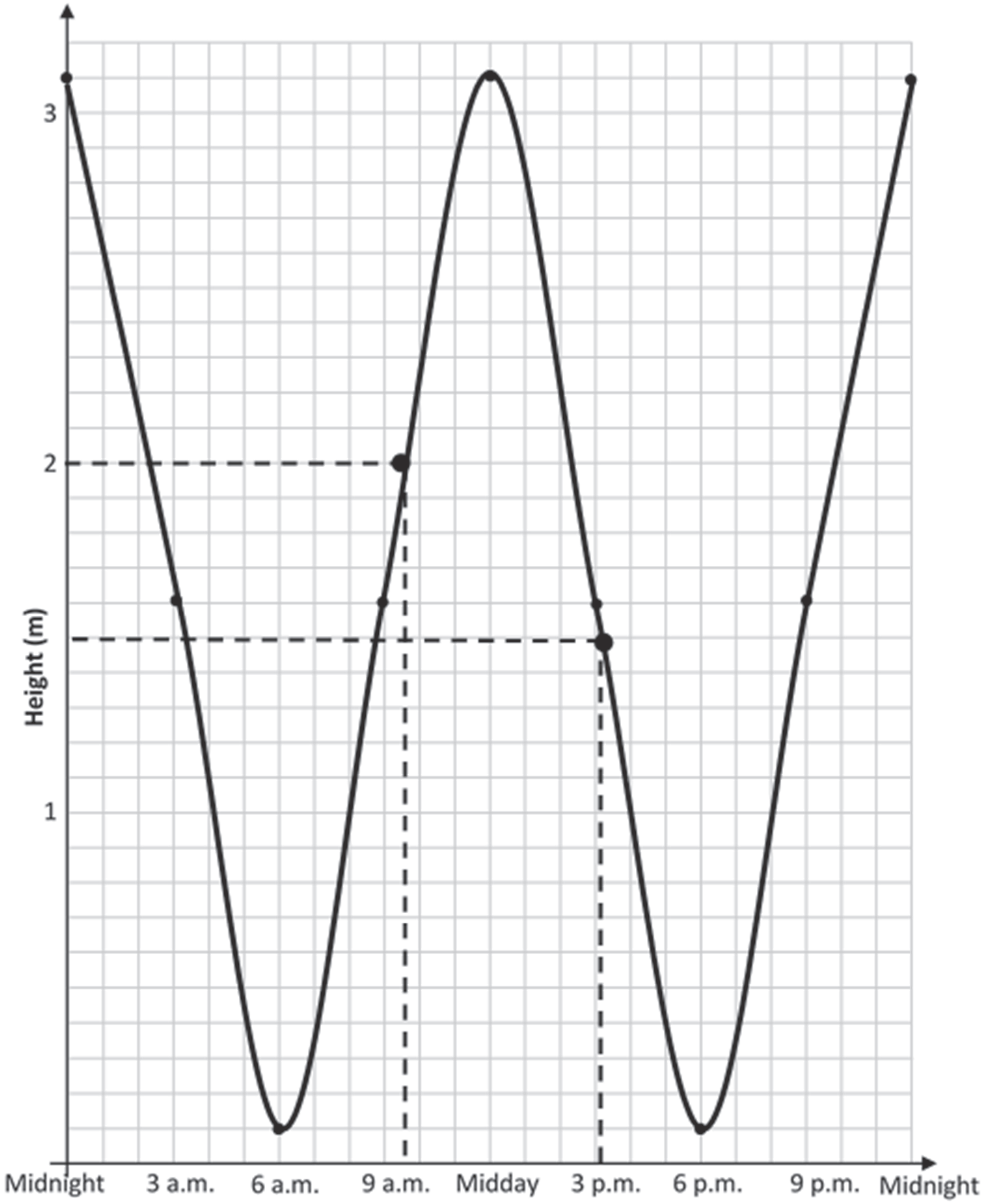
(a) **(v)**	$A = 2\times$isosceles triangle $+2\times$equilateral triangle $= 2\times\left[\frac{1}{2}(2\cdot5)(3)\sin 65°\right] +$ $2\times\left[\frac{1}{2}(3)(3)\sin 60°\right]$ $= 14\cdot59$ A=15	Scale 10D (0,3,5,8,10) *Low Partial Credit* • recognises area of 4 triangles *Mid Partial Credit* • Area of 1 triangle correct *High Partial Credit* • area of isosceles triangle and equilateral triangle **Note**: Area = 4 isosceles or 4 equilateral triangles merit *HPC* at most
(b)	$\tan 60° = \frac{3}{\lvert CA\rvert}$ $\Rightarrow \lvert CA\rvert = \sqrt{3}$ $\lvert CE\rvert = 2\sqrt{3}$ $x^2 + x^2 = \left(2\sqrt{3}\right)^2$ $x = \sqrt{6}$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • effort at Pythagoras but without $\lvert CA\rvert$ (or $\lvert CE\rvert$) • $\lvert CA\rvert$ found *High Partial Credit* • $\lvert CE\rvert = 2\sqrt{3}$

Q8	Model Solution – 45 Marks	Marking Notes
(a)	Period = $\dfrac{2\pi}{\frac{\pi}{6}}$ = 12 hours Range = $[1{\cdot}6 - 1{\cdot}5\,, 1{\cdot}6 + 1{\cdot}5\,] = [0{\cdot}1 \text{ m}, 3{\cdot}1 \text{ m}]$	Scale 5C (0, 2,4, 5) *Low Partial Credit* • some use of 2π or $\dfrac{\pi}{6}$ • range of cos function *High partial credit* • period or range correct **Note:** *Accept correct period and/or range without work*
(b)	Max = 1·6 + 1·5(1) =3·1 m. **or** 3·1 m from range	Scale 5B (0,2, 5) *Partial Credit* • max occurs when cos *A* = 1 or *t* = 0 • effort at $h'(t)$ **Note:** *Accept correct answer without work*
(c)	$h'(t) = 1{\cdot}5(-\sin\dfrac{\pi t}{6})\dfrac{\pi}{6}$ $h'(2) = 1{\cdot}5(-\sin\dfrac{2\pi}{6})\dfrac{\pi}{6}$ = – 0·68017 =$-0{\cdot}68$ m/h Tide is going out at a rate of $0{\cdot}68$ m per hour at 2 am	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • effort at differentiation *High Partial Credit* • correct numerical answer but not in context

(d)(i)

$h(t) = 1 \cdot 6 + 1 \cdot 5 \cos\left(\frac{\pi}{6}t\right)$									
Time	12 am	3 am	6 am	9 am	12 pm	3 pm	6 pm	9 pm	12 am
t	0	3	6	9	12	15	18	21	24
Height	3·1	1·6	·1	1·6	3·1	1·6	·1	1·6	3·1

(d) **(i)**		Scale 10C (0, 3, 7, 10) *Low Partial Credit* • one correct height *High Partial Credit* • five correct heights

(d) **(ii)**	

(d) **(ii)**	Graph	Scale 10C (0, 3,7,10) *Low Partial Credit* • one correct plot *High Partial Credit* • at least 7 correct plots • plots correct but graph not sketched or sketched incorrectly

(e)	Low tide = $0 \cdot 1$ m High tide = 3·1 m Difference = 3·1 — 0·1 =3 m	Scale 5B (0, 2, 5) *Partial Credit* • height of Low tide or High tide correctly identified **Notes:** (i) *candidates may show work for this section on graph* (ii) *accept values from candidate's graph* (iii) *accept correct answer from graph without work*
(f)	Enter port at 9:30 approx Leave port before 15:15 approx Time = 15:15 – 9:30 = 5 hr 45 min approx.	Scale 5B (0, 2, 5) *Partial Credit* • time of entry to port or leave port correctly identified • value(s) for h = 2 and/or h = 1·5 on sketch • time estimated using relevant values other than those required for the maximum time. **Notes:** (i) *candidates may show relevant work for this section on graph* (ii) *accept values from candidate's graph*

Q9	Model Solution – 50 Marks	Marking Notes
(a) **(i)**	$\mu = 39400, \quad \sigma = 12920$ $z = \dfrac{x-\mu}{\sigma} = \dfrac{60000 - 39400}{12920}$ $z = 1{\cdot}59$ $P(z > 1{\cdot}59) = 1 - P(z < 1{\cdot}59)$ $= 1 - 0{\cdot}9441 = 0{\cdot}0559$ $= 5{\cdot}59\%$ = 5·6%	Scale 10D (0, 3, 5, 8, 10) *Low Partial Credit* • μ and σ identified *Mid Partial Credit* • z = 1·59 *High Partial Credit* • identifies 0·9441
(a) **(ii)**	$P(z \leq z_1) = 0{\cdot}9$ $z_1 = 1{\cdot}28$ $\Rightarrow z_2 = -1{\cdot}28$ $\Rightarrow \dfrac{x - 39400}{12920} = -1{\cdot}28$ $x = 22862{\cdot}40$ = €22 862	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • identifies 1·28 but fails to progress *High Partial Credit* • formula for x fully substituted
(a) **(iii)**	$\mu = 39400, \quad \sigma = 12920,$ $\bar{x} = 38280, \quad n = 1000$ $H_0 \Rightarrow \mu = 39400$ $H_1 \Rightarrow \mu \neq 39400$ $z = \dfrac{38280 - 39400}{\frac{12920}{\sqrt{1000}}} = -2{\cdot}74$ $-2{\cdot}74 < -1{\cdot}96$ Result is significant. There is evidence to reject the null hypothesis The mean income has changed.	Scale 15D (0, 4, 7, 11,15) *Low Partial Credit* • z formulated with some substitution • states null and/or alternative hypothesis only • reference to 1·96 Mid Partial Credit • z fully substituted *High Partial Credit* • $z = -2{\cdot}74$ and stops • fails to state the null and alternative hypothesis correctly • fails to contextualise the answer

	or Confidence Interval: $$\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$$ $$39400 \pm 1{\cdot}96\frac{12920}{\sqrt{1000}}$$ $$[38599{\cdot}2,\ 40200{\cdot}8]$$ 38280 outside range Result is significant. There is evidence to reject the null hypothesis The mean income has changed. **or** Confidence Interval: $$\bar{x} \pm 1{\cdot}96\frac{\sigma}{\sqrt{n}}$$ $$38280 \pm 1{\cdot}96\frac{12920}{\sqrt{1000}}$$ $$38280 \pm 1{\cdot}96(408{\cdot}57)$$ $$[37479{\cdot}2,\ \ 39080{\cdot}8]$$ 39400 outside range Result is significant. There is evidence to reject the null hypothesis The mean income has changed.	

Q9		Marking Notes
(b)	$26974 - 1{\cdot}96\left(\frac{5120}{\sqrt{400}}\right) \leq \mu$ $\leq 26974 + 1{\cdot}96\left(\frac{5120}{\sqrt{400}}\right)$ $26472{\cdot}24 \leq \mu \leq 27475{\cdot}76$	Scale 10C (0, 3, 7, 10) *Low Partial Credit* • interval formulated with some correct substitution *High Partial Credit* • interval formulated with fully correct substitution
(c)	The distribution of sample means will be normally distributed	Scale 5B (0, 2, 5) *Partial Credit* • mentions 30 (or more) but not contextualised
(d)	$\frac{1}{\sqrt{n}} = 0{\cdot}045$ $\frac{1}{0{\cdot}045} = \sqrt{n}$ $n = \left(\frac{1}{0{\cdot}045}\right)^2 = 493{\cdot}827$	Scale 5C (0, 2, 4, 5) *Low Partial Credit* • $\frac{1}{\sqrt{n}}$ *High Partial Credit* • n formulated with fully correct substitution **Note:** Accept 493 farmers or 494 farmers

Marcanna breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn níos mó ná 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéir ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás iarrthóirí a ghnóthaíonn 225 marc nó níos lú, e.g. 198 marc $\times$ 5% = 9·9 $\Rightarrow$ bónas = 9 marc.

Má ghnóthaíonn an t-iarrthóir níos mó ná 225 marc, ríomhtar an bónas de réir na foirmle [300 – bunmharc] $\times$ 15%, agus an marc bónais sin a shlánú **síos**. In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0

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